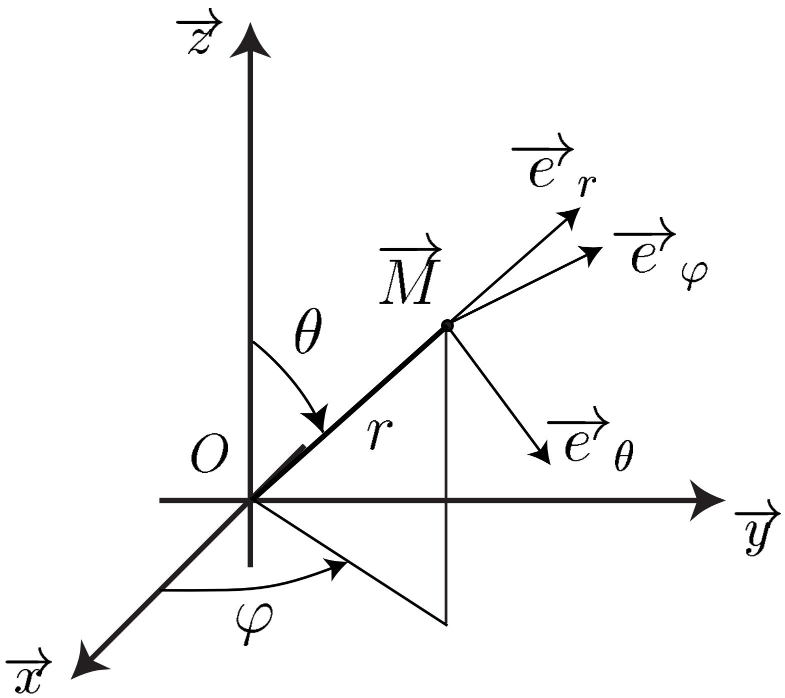
Annexe B

Coordonnées sphériques



Fonction scalaire

$$f(\overrightarrow{M}) = f(r,\, \theta,\, \varphi)$$

Gradient

$$\overrightarrow{\text{grad}}(f) = \begin{bmatrix} \dfrac{\partial f}{\partial r} \\ \dfrac{1}{r}\dfrac{\partial f}{\partial \theta} \\ \dfrac{1}{r\,\sin\theta}\dfrac{\partial f}{\partial \varphi} \end{bmatrix}_{(\overrightarrow{e}_r,\, \overrightarrow{e}_\theta,\, \overrightarrow{e}_\varphi)}$$

Champs de vecteur

$$\overrightarrow{V}(\overrightarrow{M}) = V_r(r,\, \theta,\, \varphi)\, \overrightarrow{e}_r + V_\theta(r,\, \theta,\, \varphi)\, \overrightarrow{e}_\theta + V_\varphi(r,\, \theta,\, \varphi)\, \overrightarrow{e}_\varphi$$

Gradient

$$\mathbb{G}\text{rad}(\overrightarrow{V}) = \begin{bmatrix} \dfrac{\partial V_r}{\partial r} & \dfrac{1}{r}\left(\dfrac{\partial V_r}{\partial \theta} - V_\theta\right) & \dfrac{1}{r}\left(\dfrac{1}{\sin\theta}\dfrac{\partial V_r}{\partial \varphi} - V_\varphi\right) \\ \dfrac{\partial V_\theta}{\partial r} & \dfrac{1}{r}\left(\dfrac{\partial V_\theta}{\partial \theta} + V_r\right) & \dfrac{1}{r}\left(\dfrac{1}{\sin\theta}\dfrac{\partial V_\theta}{\partial \varphi} - \dfrac{1}{\tan\theta}V_\varphi\right) \\ \dfrac{\partial V_\varphi}{\partial r} & \dfrac{1}{r}\dfrac{\partial V_\varphi}{\partial \theta} & \dfrac{1}{r}\left(\dfrac{1}{\sin\theta}\dfrac{\partial V_\varphi}{\partial \varphi} + \dfrac{1}{\tan\theta}V_\theta + V_r\right) \end{bmatrix}_{(\overrightarrow{e}_r,\, \overrightarrow{e}_\theta,\, \overrightarrow{e}_\varphi)}$$

Divergence

$$\mathrm{div}(\overrightarrow{V}) = \frac{\partial V_r}{\partial r} + \frac{1}{r}\frac{\partial V_\theta}{\partial \theta} + \frac{1}{r\,\sin\theta}\frac{\partial V_\varphi}{\partial \varphi} + \frac{1}{r\,\tan\theta}V_\theta + 2\,\frac{V_r}{r}$$

Champ de tenseur du 2$^{\mathrm{e}}$ ordre (symétrique)

$$\mathbb{A}(\overrightarrow{M}) = \begin{bmatrix} A_{rr}(r,\,\theta,\,\varphi) & A_{r\theta}(r,\,\theta,\,\varphi) & A_{r\varphi}(r,\,\theta,\,\varphi) \\ \vdots & A_{\theta\theta}(r,\,\theta,\,\varphi) & A_{\theta\varphi}(r,\,\theta,\,\varphi) \\ sym & \cdots & A_{zz}(r,\,\theta,\,\varphi) \end{bmatrix}_{(\overrightarrow{e}_r,\,\overrightarrow{e}_\theta,\,\overrightarrow{e}_\varphi)}$$

Divergence

$$\overrightarrow{\mathrm{div}}(\mathbb{A}) = \begin{bmatrix} \dfrac{\partial A_{rr}}{\partial r} + \dfrac{1}{r}\dfrac{\partial A_{r\theta}}{\partial \theta} + \dfrac{1}{r\,\sin\theta}\dfrac{\partial A_{r\varphi}}{\partial \varphi} + \dfrac{1}{r}\left(2\,A_{rr} - A_{\theta\theta} - A_{\varphi\varphi} + \dfrac{1}{\tan\theta}A_{r\theta}\right) \\ \dfrac{\partial A_{r\theta}}{\partial r} + \dfrac{1}{r}\dfrac{\partial A_{\theta\theta}}{\partial \theta} + \dfrac{1}{r\,\sin\theta}\dfrac{\partial A_{\theta\varphi}}{\partial \varphi} + \dfrac{1}{r}\left(\dfrac{A_{\theta\theta} - A_{\varphi\varphi}}{\tan\theta} + 3\,A_{r\theta}\right) \\ \dfrac{\partial A_{r\varphi}}{\partial r} + \dfrac{1}{r}\dfrac{\partial A_{\theta\varphi}}{\partial \theta} + \dfrac{1}{r\,\sin\theta}\dfrac{\partial A_{\varphi\varphi}}{\partial \varphi} + \dfrac{1}{r}\left(2\,\dfrac{A_{\theta\varphi}}{\tan\theta} + 3\,A_{r\varphi}\right) \end{bmatrix}_{(\overrightarrow{e}_r,\,\overrightarrow{e}_\theta,\,\overrightarrow{e}_\varphi)}$$