Stochastic B–series and order conditions for exponential integrators

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We will discuss B–series for the solution of a stochastic differential equation of the form

$$\mathrm{d}X(t) = \Big(AX(t) + g_0\big(X(t)\big)\Big)\mathrm{d}t + \sum_{m=1}^{M} g_m(X(t)) \star \mathrm{d}W_m(t), \quad X(0) = x_0,$$

for which the exact solution can be written as

$$X(t) = e^{tA}x_0 + \int_0^t e^{(t-s)A} g_0(X(s))\mathrm{d}s + \sum_{m=1}^{M} \int_0^t e^{(t-s)A} g_m(X(s)) \star \mathrm{d}W_m(s).$$

Based on this, we will derive an order theory for exponential integrators for such problems. We will then discuss how the order theory for exponential integrators derived can be simplified and adjusted to a semilinear SDE with time-dependent additive noise, and present a mean square order 1.5 method based on these conditions. The method has been applied to a semi-discretized diffusion-reaction PDE with bounday noise, and some implementation issues will be discussed.

References

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