Une modélisation de la mécanique des mousses

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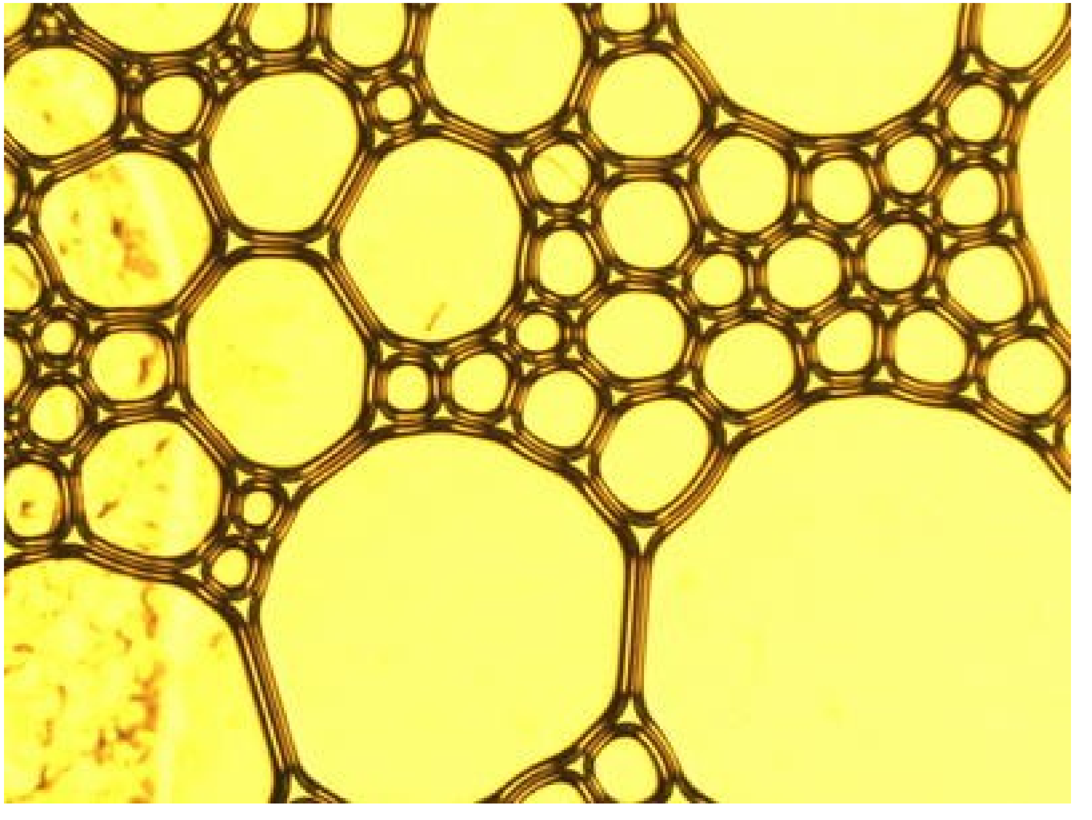


Réponse mécanique d'un milieu mou

(wikipédia : rhéologie)

Faibles contraintes

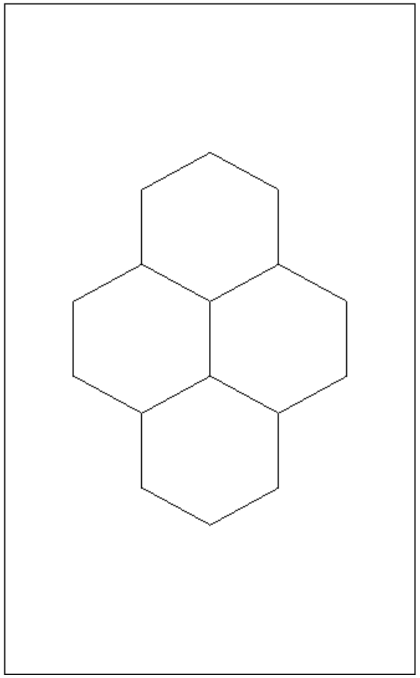
Élastique vs. visqueux

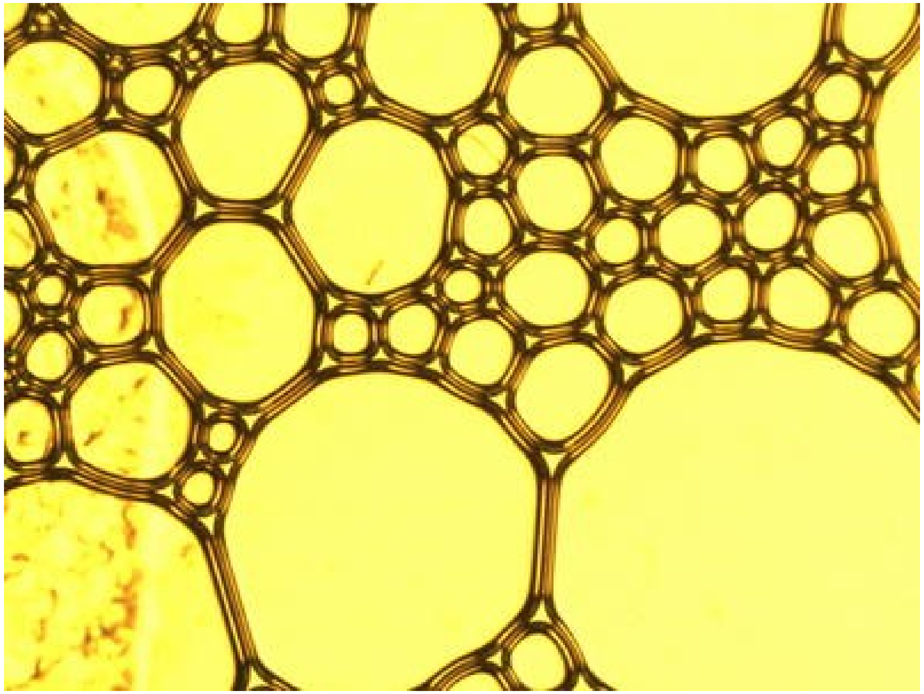
Liquide vs. solide

Fortes contraintes

Écoulement d'un solide : plasticité

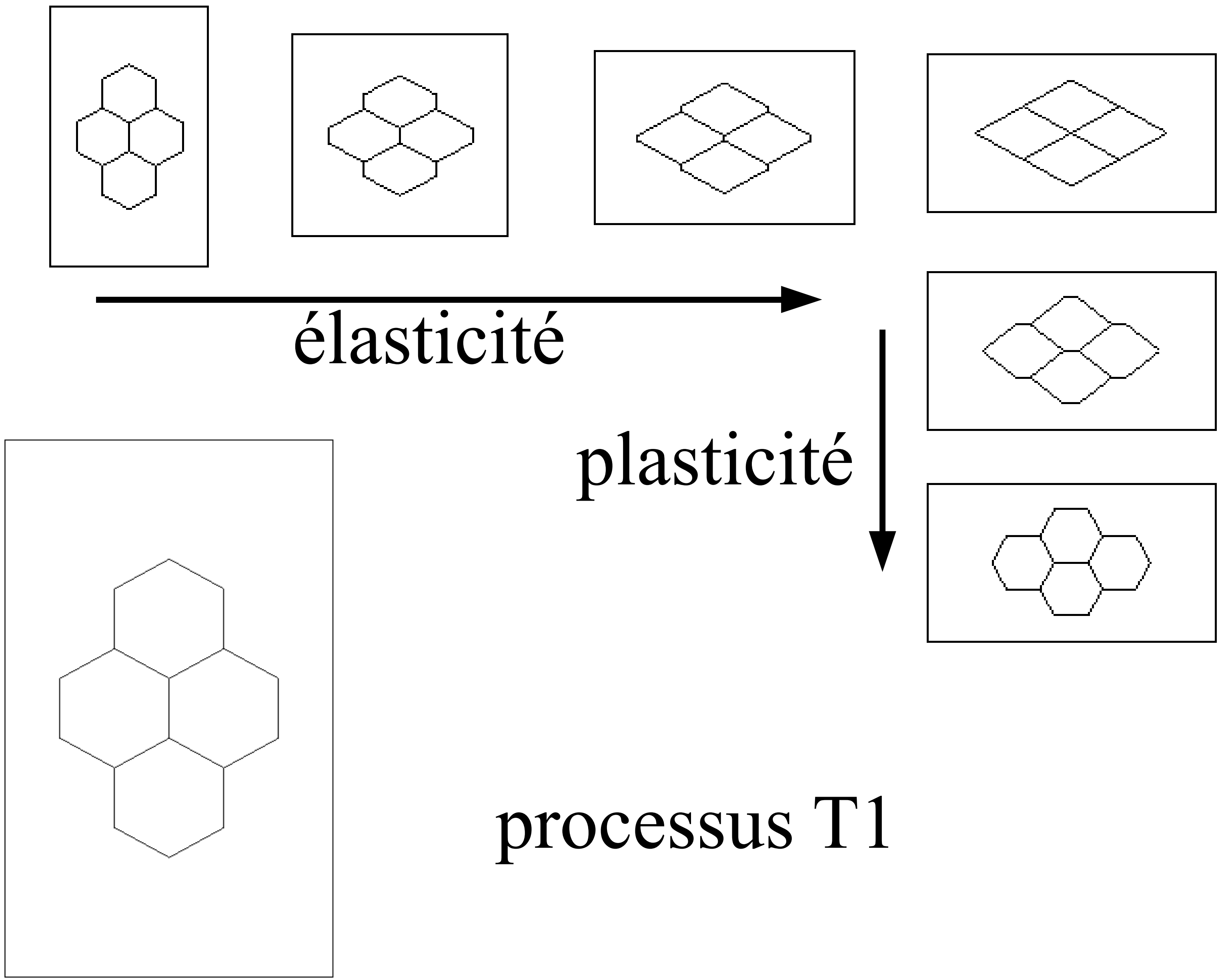
Une mousse





processus T1

Une mousse



Plan

Mémoire du matériau et système de coordonnées

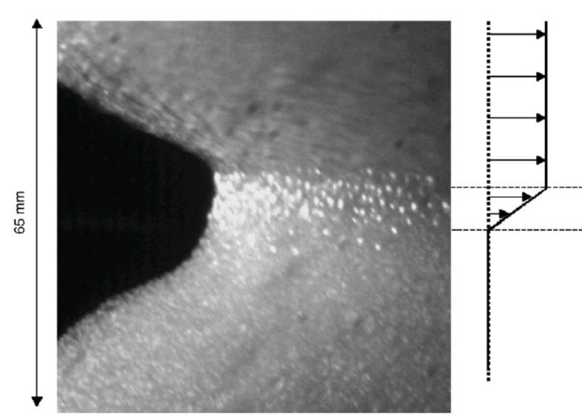
Élasticité :

définitions de la déformation

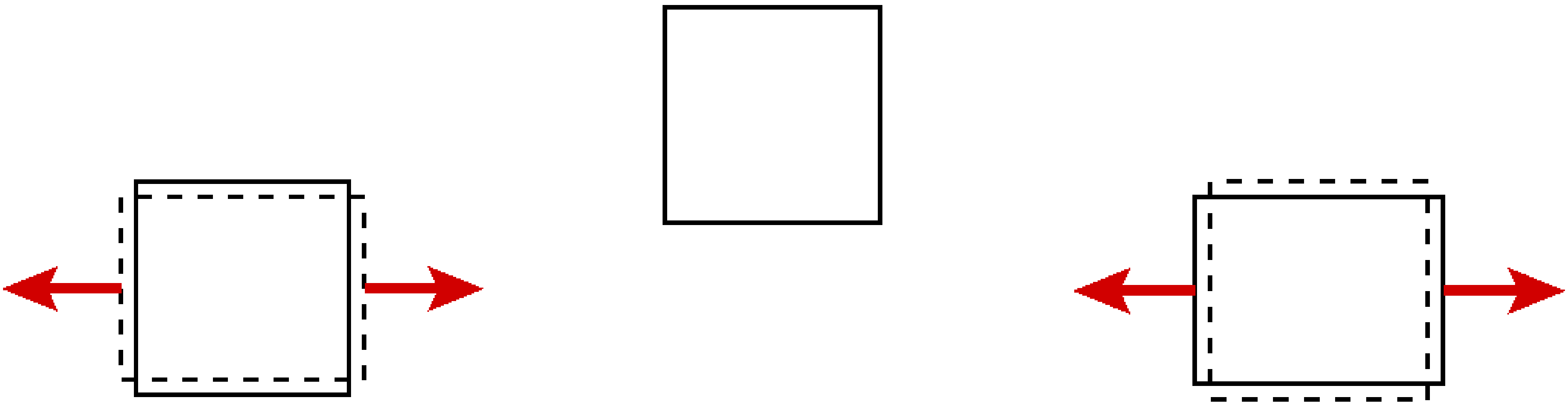
lois admissibles d'évolution de la contrainte

Plasticité, viscosité, modèle complet

Dilatance



Grandes déformations



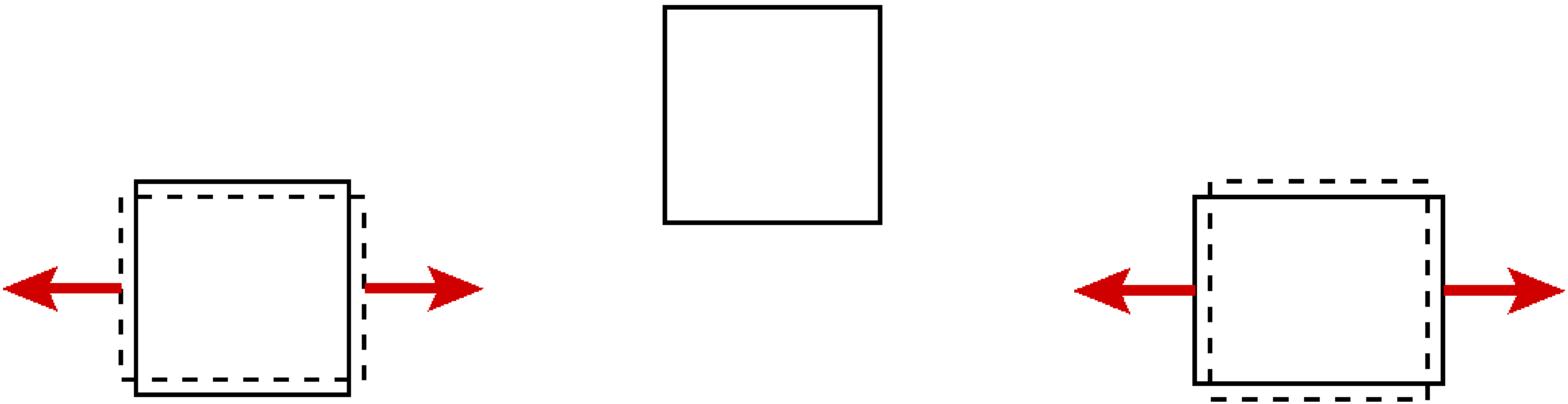
$$\sigma_{\text{nominale}} = F/S_0$$

Contrainte nominale

$$\sigma_{\text{vraie}} = F/S(t)$$

Contrainte vraie

Grandes déformations



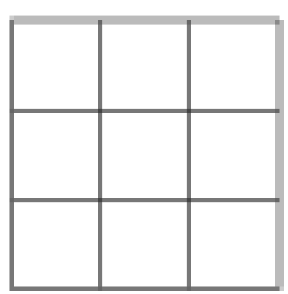
$$\sigma_{\text{nominale}} = F/S_0$$

Contrainte nominale

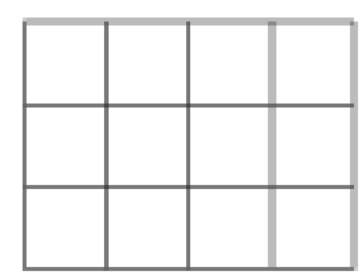
$$\sigma_{\text{vraie}} = F/S(t)$$

Contrainte vraie

Approche lagrangienne

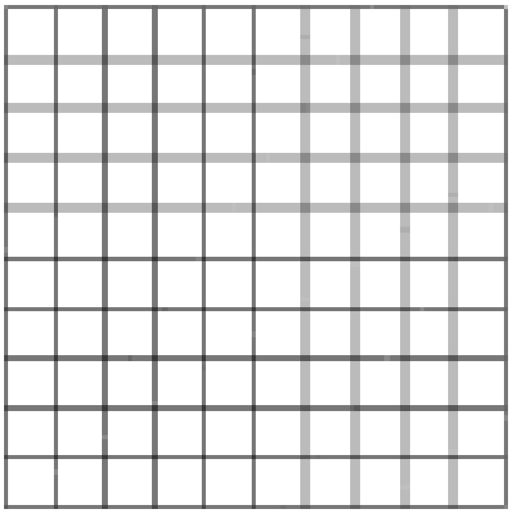


Approche eulérienne



Grandes déformations

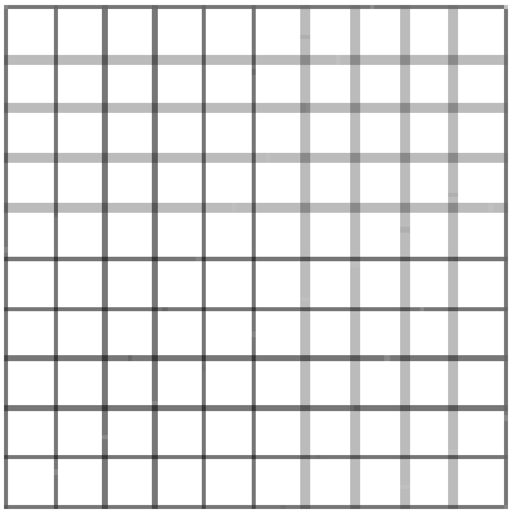
Élasticité



mémoire de l'état initial

Grandes déformations

Élasticité



mémoire de l'état initial

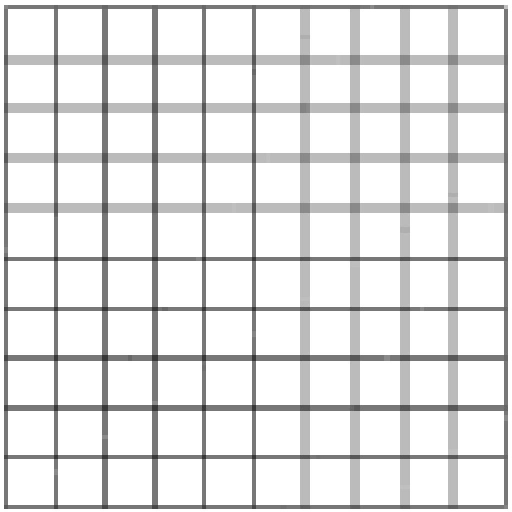
Approche lagrangienne

Approche eulérienne

(Landau)

Grandes déformations

Élasticité



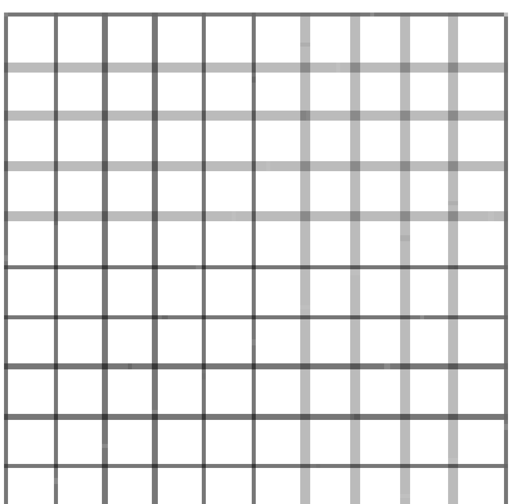
mémoire de l'état initial

Approche lagrangienne

Approche eulérienne

(Landau)

Mécanique des fluides



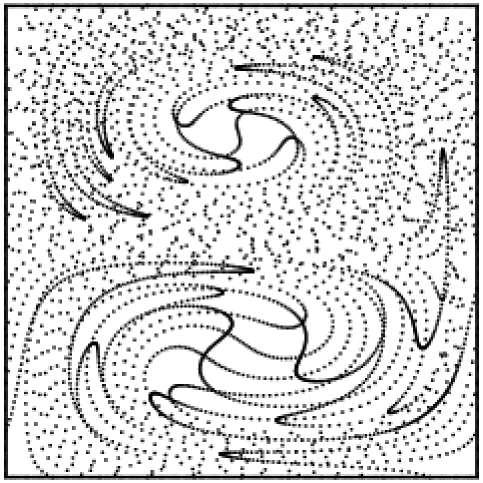
matériau sans mémoire

Approche eulérienne

(Navier-Stokes)

Grandes déformations

Et les mousses ?



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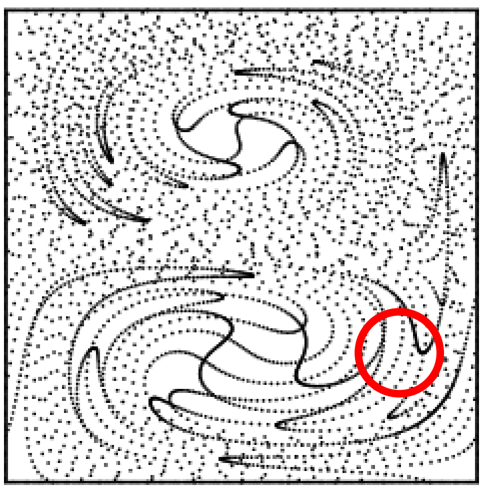
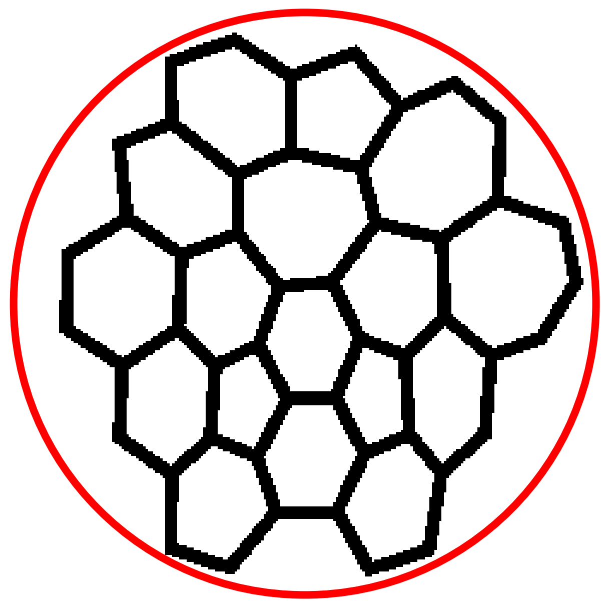
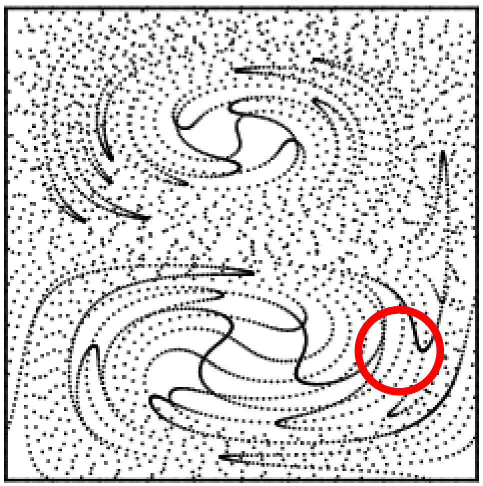




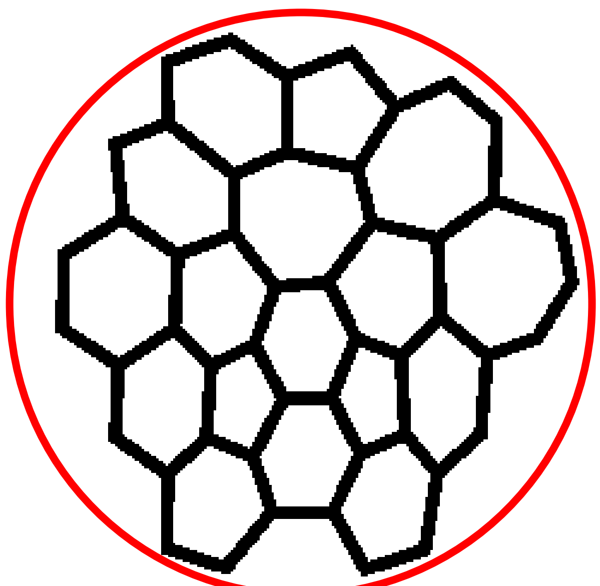
Grandes déformations

Et les mousses ?



sans
mémoire

T1



Approche
eulérienne

Matériau sans mémoire :

fluide
newtonien

mousses

Approche
lagrangienne !

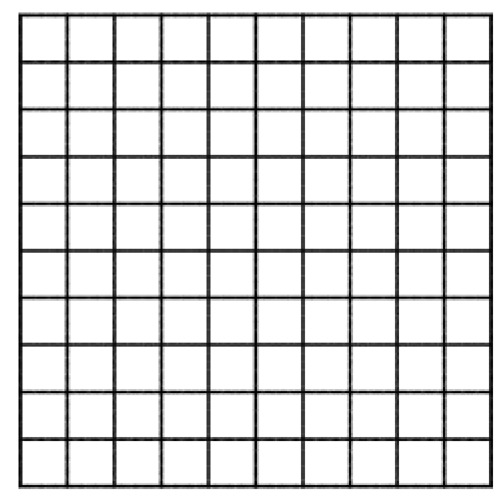
temps t

Matériau sans mémoire :

Approche eulérienne !

fluide newtonien

mousses



temps t

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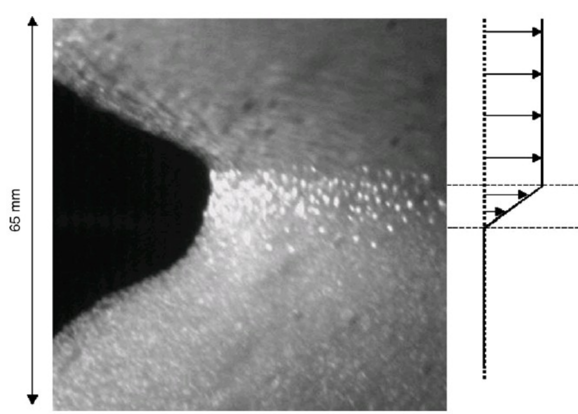
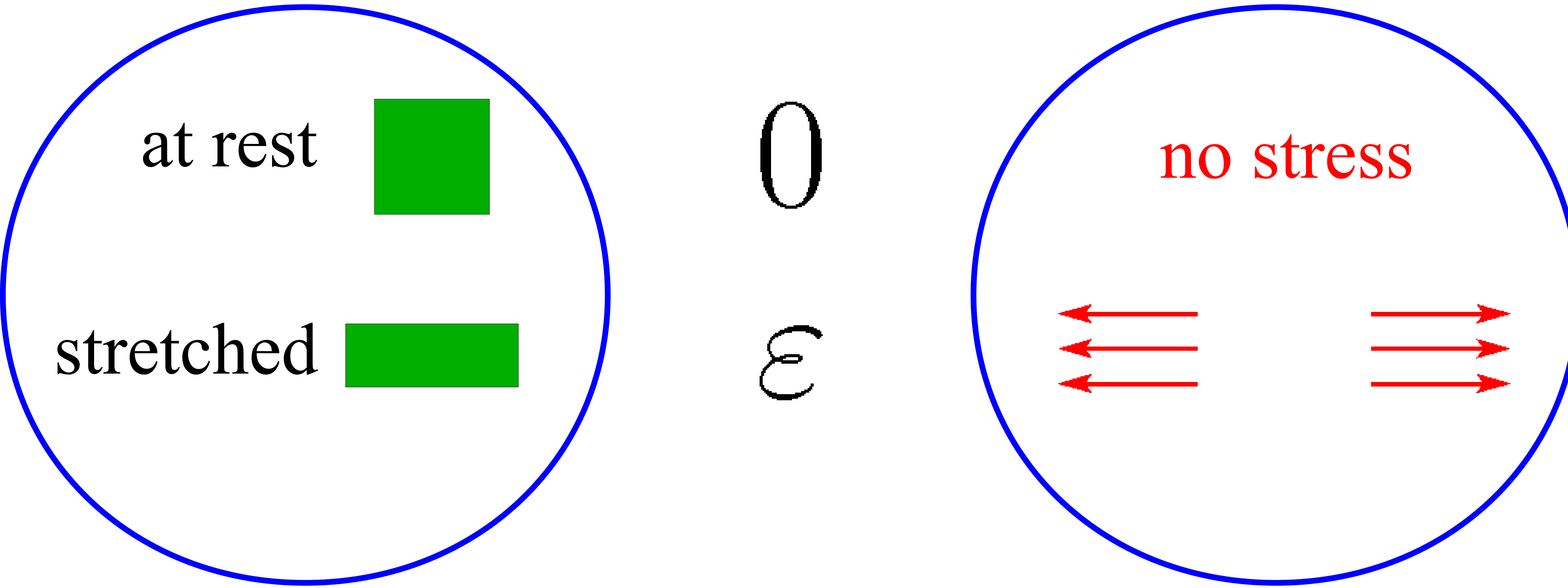
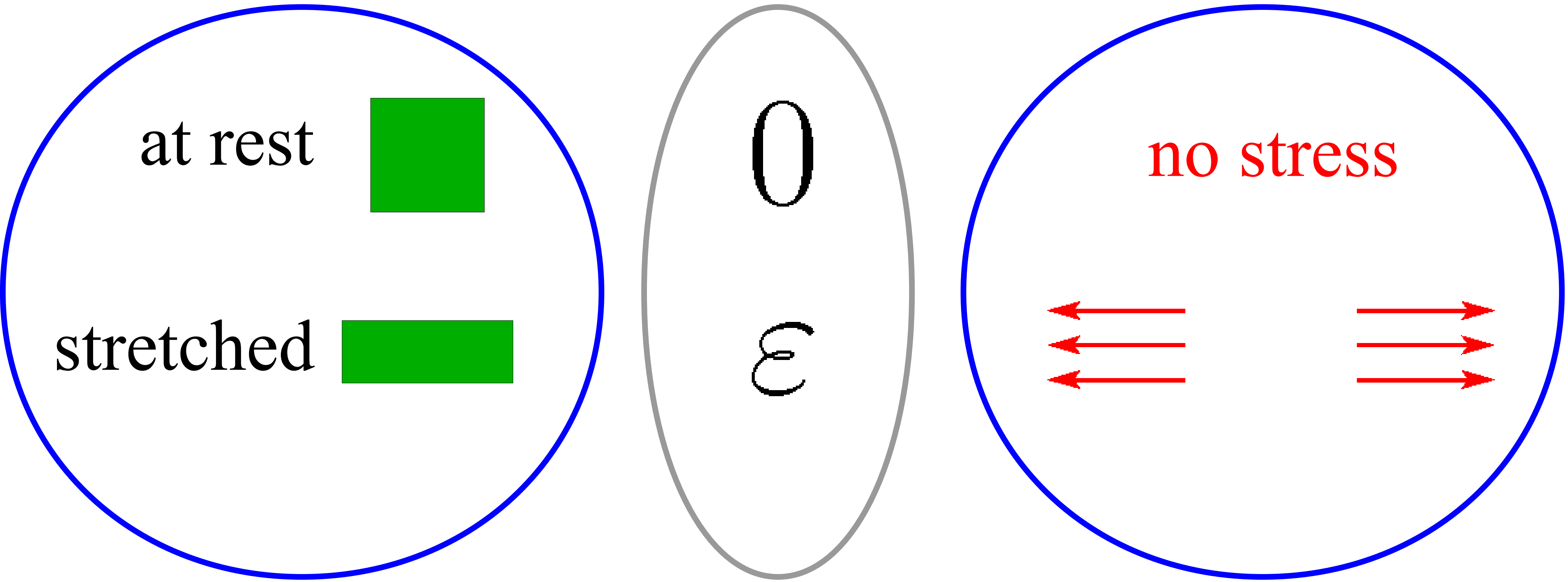


State - deformation - stress



physics
(intrinsic)

State - deformation - stress



physics
(intrinsic)

description
(conventional:
many deformations)

Many deformations

All deformations coincide at small deformations

An elastic law is a relation : $e \leftrightarrow \sigma$

Many possible choices for the deformation :

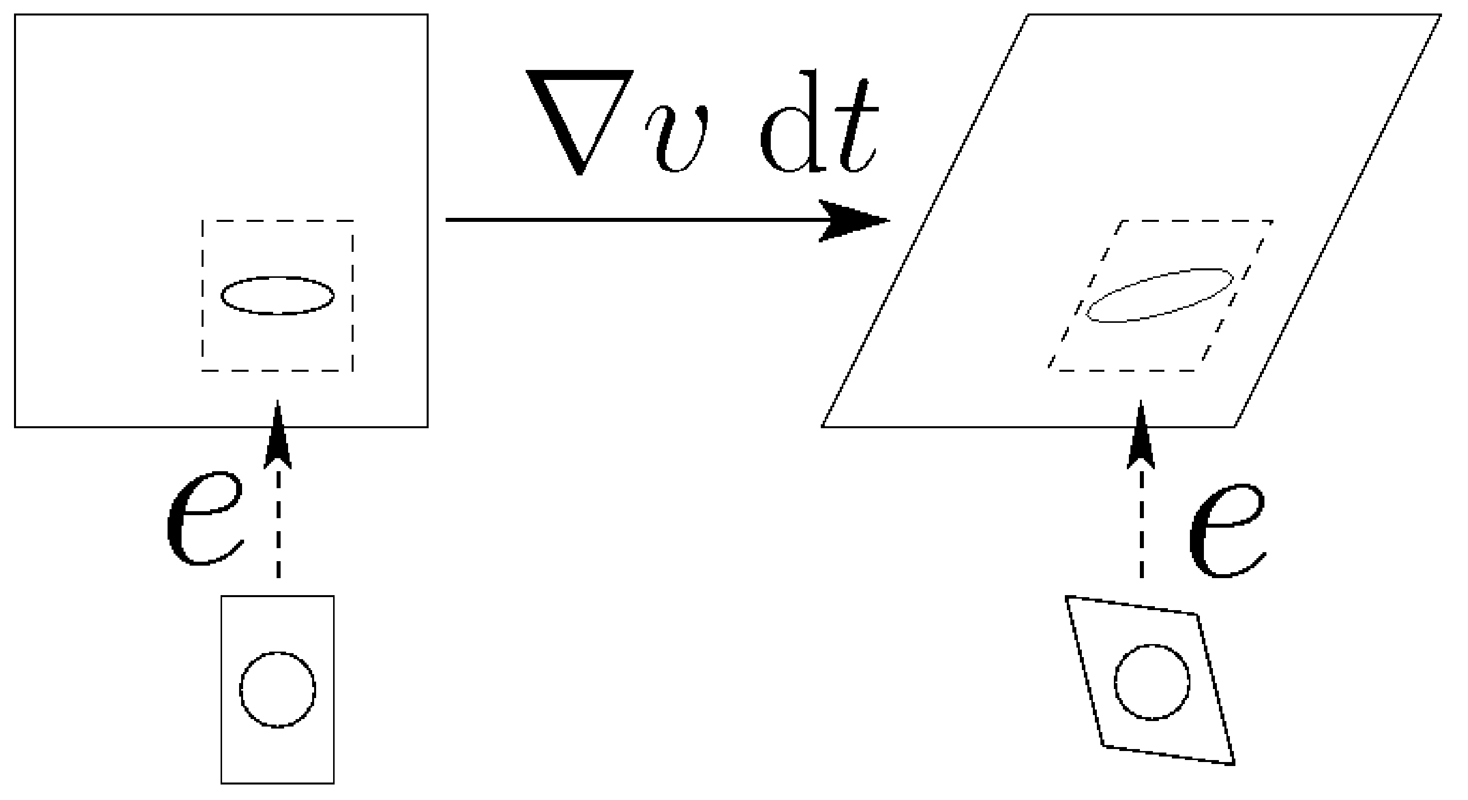
simple elastic law

simple kinematics

...

Cinématique simple

$$\frac{\mathrm{D}e}{\mathrm{D}t} = f\left(\nabla v,\ e\right)$$



$$\frac{\mathrm{D}e}{\mathrm{D}t} = \frac{\partial e}{\partial t} + (v \cdot \nabla) e - \nabla v \cdot e - e \cdot \nabla v^{\mathrm{T}}$$

Evolution

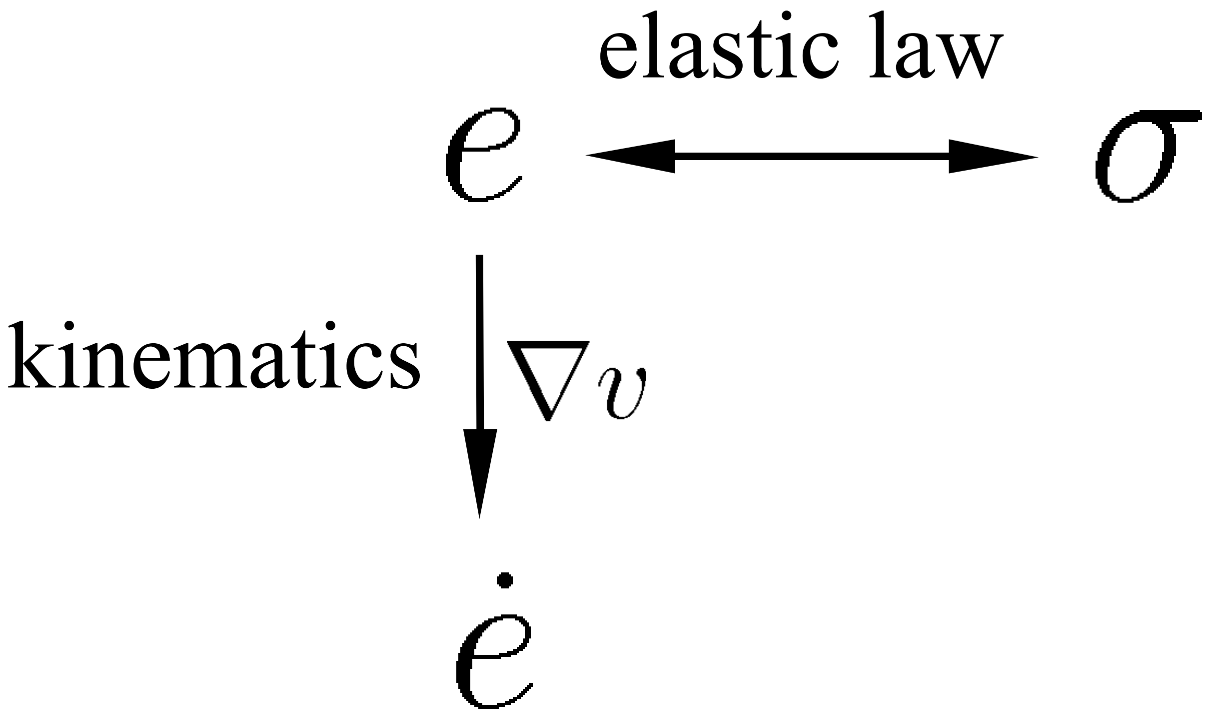
$$e$$

kinematics $\Big\downarrow \nabla v$

$$\dot{e}$$

$$\dot{e} = \dot{e}\left(\nabla v,\ e\right)$$

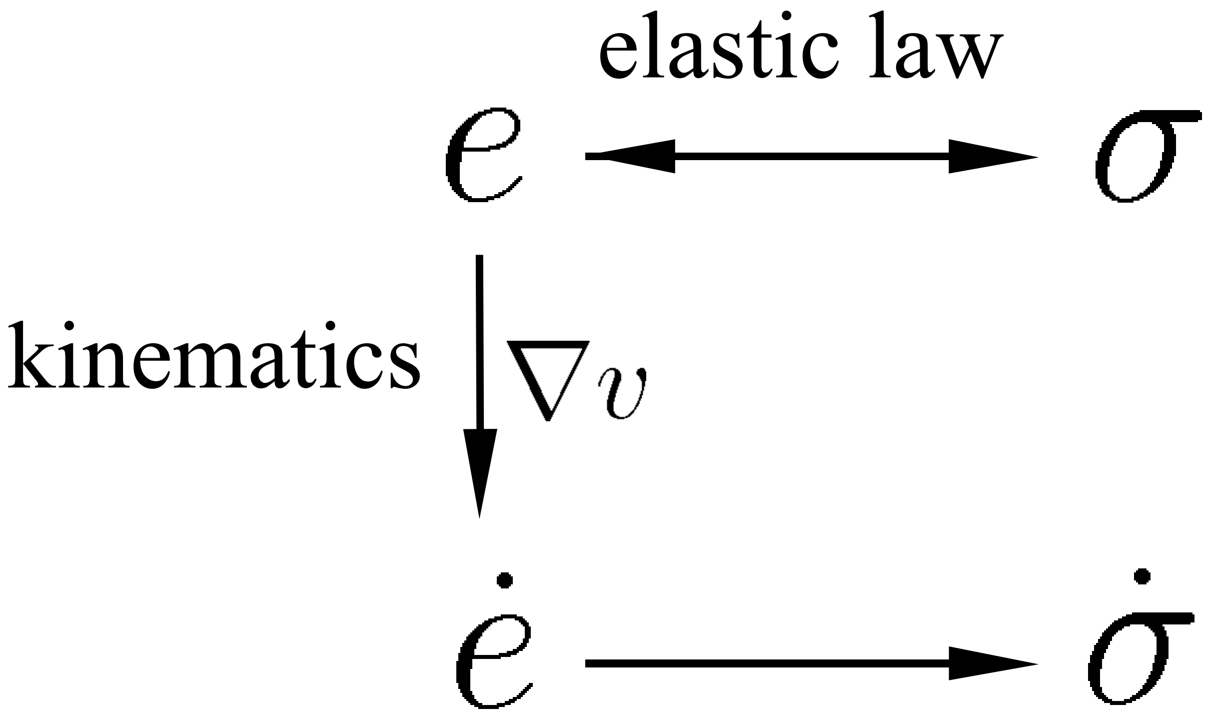
Evolution



$$\sigma = \sigma\left(e\right)$$

$$\dot{e} = \dot{e}\left(\nabla v,\ e\right)$$

Evolution

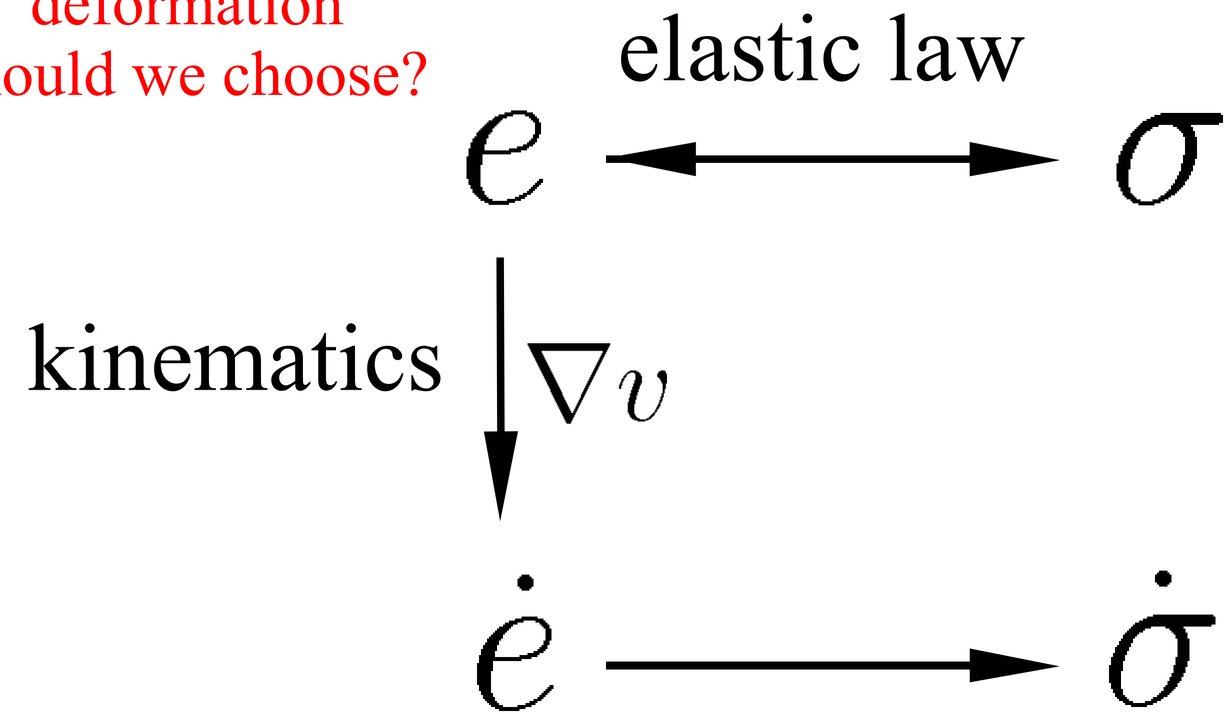


$$\dot{\sigma} = \dot{\sigma}\left(\nabla v,\ \sigma\right)$$

Bénito et al. Eur. Phys. J. E 2008

Evolution

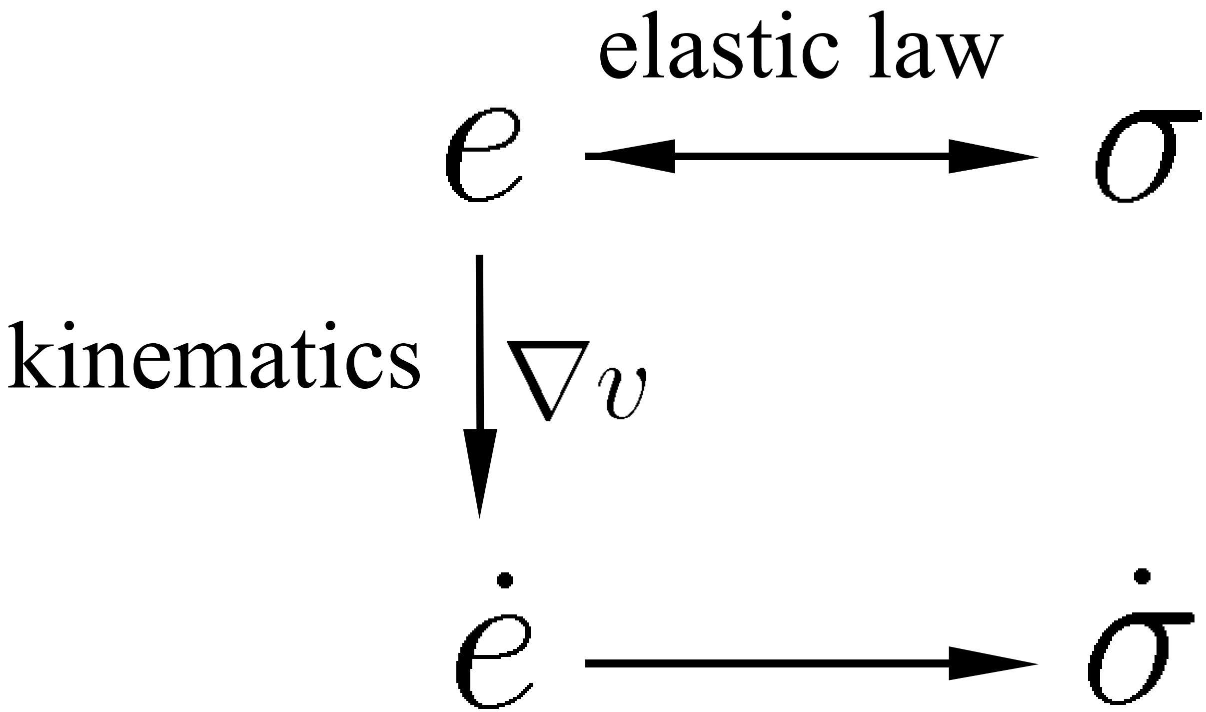
Which deformation should we choose?



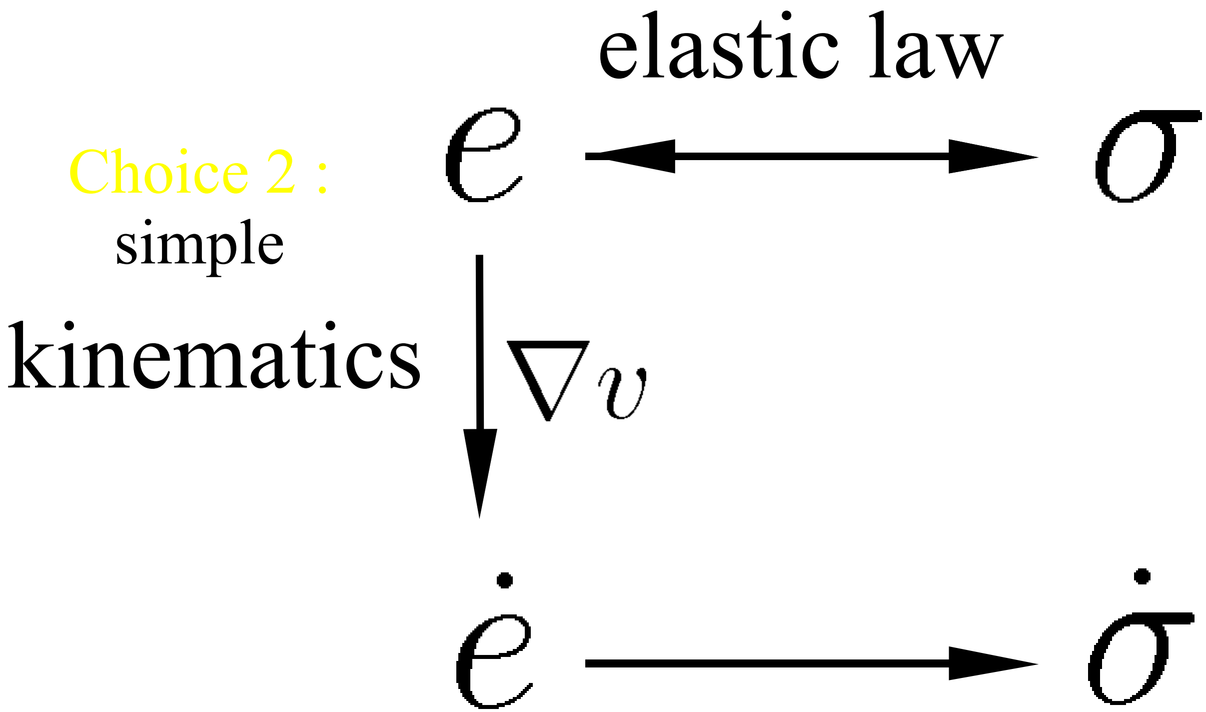
$$\dot{\sigma} = \dot{\sigma}\left(\nabla v,\ \sigma\right)$$

Choice

Choice 1 : simple



Choice


elastic law
e
σ
Choice 2 :
simple
kinematics
∇v
$\dot{e}$
$\dot{\sigma}$

Is my favourite rheological law valid?

$$\sigma$$

∇v ↓ my favourite rheological law

$$\dot{\sigma}$$

$$\dot{\sigma} = \dot{\sigma}\left(\nabla v,\ \sigma\right)$$

intrinsic!

Is my favourite rheological law valid?

for any choice of deformation e

$$\sigma$$

∇v ↓ my favourite rheological law

$$\dot{\sigma}$$

$$\dot{\sigma} = \dot{\sigma}\left(\nabla v,\ \sigma\right)$$

intrinsic!

Is my favourite rheological law valid?

for any choice of deformation $e \longleftrightarrow \sigma$ $\qquad \sigma = \sigma\left(e\right)$

$\nabla v \big\downarrow$ my favourite rheological law

$\dot{\sigma}$

$$\dot{\sigma} = \dot{\sigma}\left(\nabla v,\ \sigma\right)$$

intrinsic!

Is my favourite rheological law valid?

for any choice of deformation

$$e \longleftrightarrow \sigma \qquad \sigma = \sigma(e)$$

$$\nabla v \Big\downarrow \text{ my favourite rheological law}$$

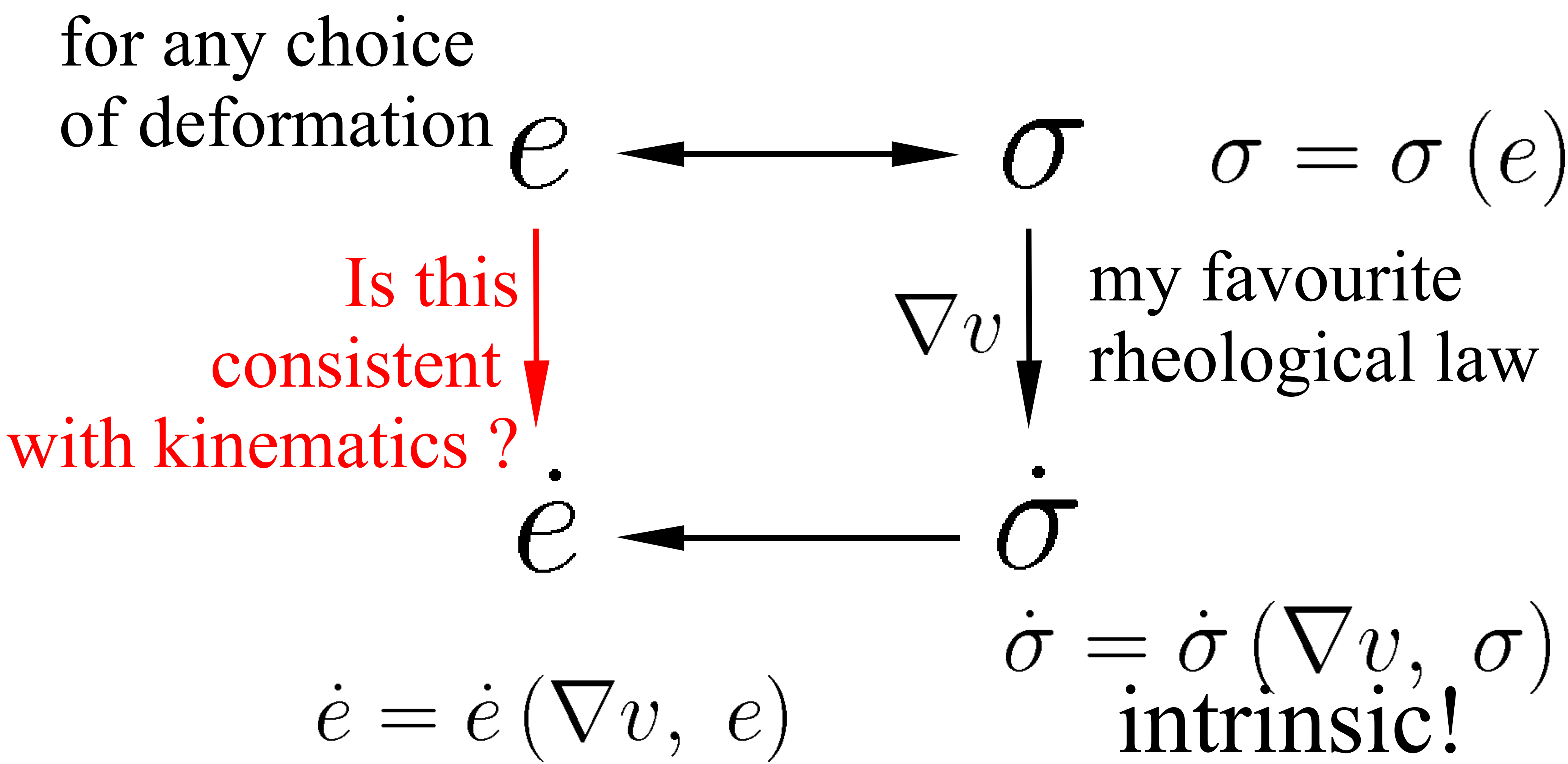
$$\dot{e} \longleftarrow \dot{\sigma}$$

$$\dot{\sigma} = \dot{\sigma}(\nabla v,\ \sigma)$$

$$\dot{e} = \dot{e}(\nabla v,\ e)$$

intrinsic!

Is my favourite rheological law valid?



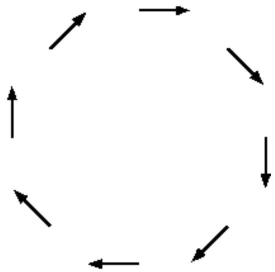
Évolution d'un tenseur

Vitesse $\nabla v = D + \Omega$

$$\Omega = \frac{\nabla v - \nabla v^T}{2}$$

$$D = \frac{\nabla v + \nabla v^T}{2}$$

Ω

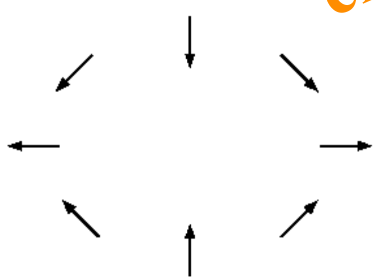


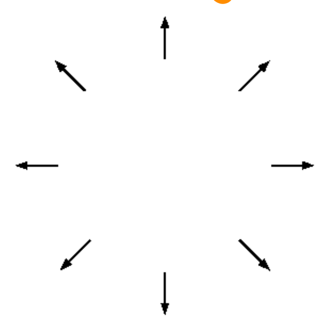
rotation

élongation

dilatation

D





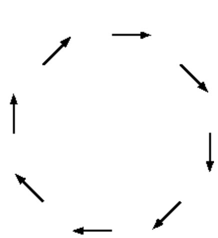
$\mathrm{tr} D = 0$ $\mathrm{tr} D \neq 0$

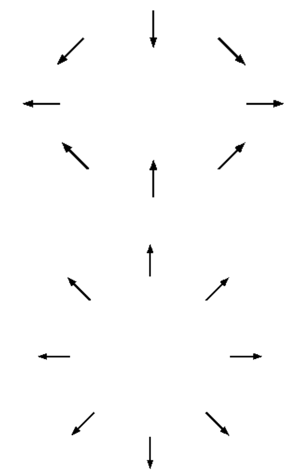
Comment évolue un tenseur symétrique ?

Évolution d'un tenseur

$$\dot{\sigma} = (\Omega\sigma - \sigma\Omega) + f(\sigma,\ D,\ \dot{D}, \ldots)$$

dérivée particulaire





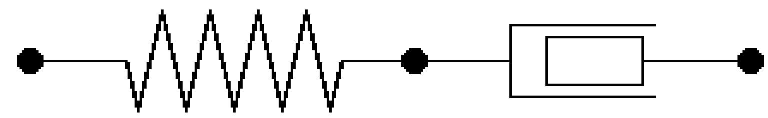
dérivée convective

autre...

équations constitutives...

Équations constitutives

Maxwell

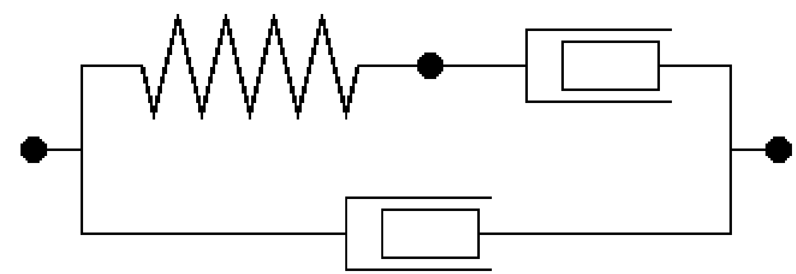


$$\dot{\sigma} = (\Omega\sigma - \sigma\Omega) + (D\sigma + \sigma D) + 2\mu D - \sigma/\tau$$

$$\dot{\sigma} = (\Omega\sigma - \sigma\Omega) - (D\sigma + \sigma D) + 2\mu D - \sigma/\tau$$

$$\dot{\sigma} = (\Omega\sigma - \sigma\Omega) \qquad\qquad\qquad\qquad + 2\mu D - \sigma/\tau$$

Jeffrey / Oldroyd



$$\sigma_{\mathrm{tot}} = \sigma + 2\eta D$$

Élasticité : $$\dot{\sigma} = (\Omega\sigma - \sigma\Omega) + f(\sigma) : D$$

Kinematic validity of some classical rheological laws

upper-convected used for some materials

$$\dot{\sigma} = 2\mu\, D + \nabla v \cdot \sigma + \sigma \cdot (\nabla v)^T$$

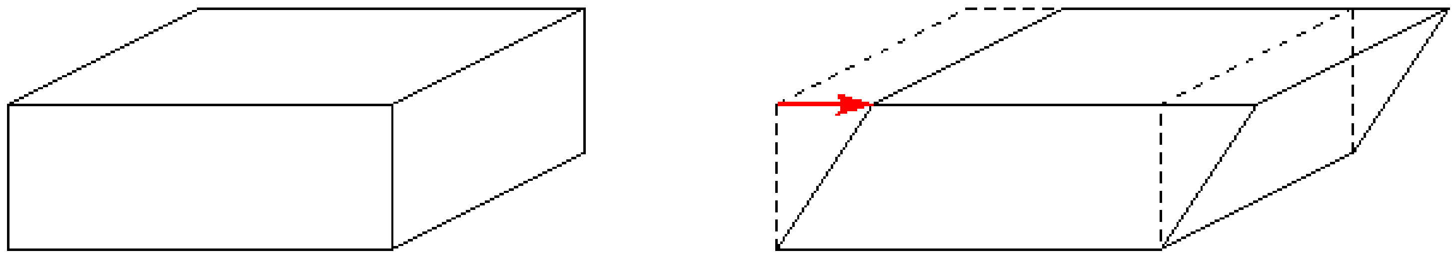
lower-convected used for some other materials

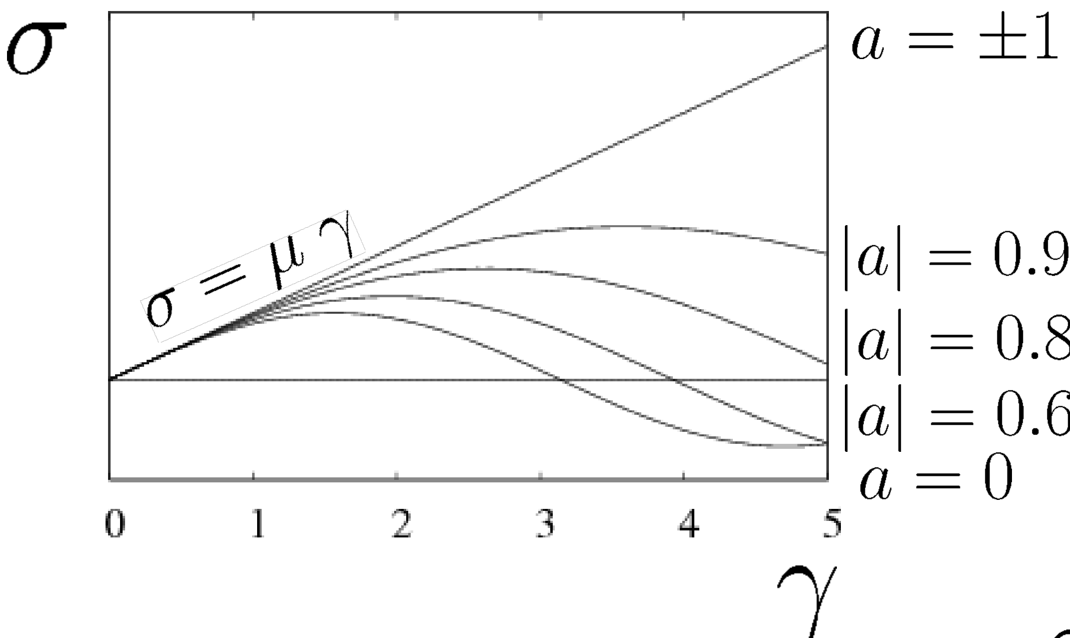
$$\dot{\sigma} = 2\mu\, D - (\nabla v)^T \cdot \sigma - \sigma \cdot \nabla v$$

corotational used for yet other materials

$$\dot{\sigma} = 2\mu\, D + \dot{\omega} \cdot \sigma - \sigma \cdot \dot{\omega}$$

Loi élastique ?

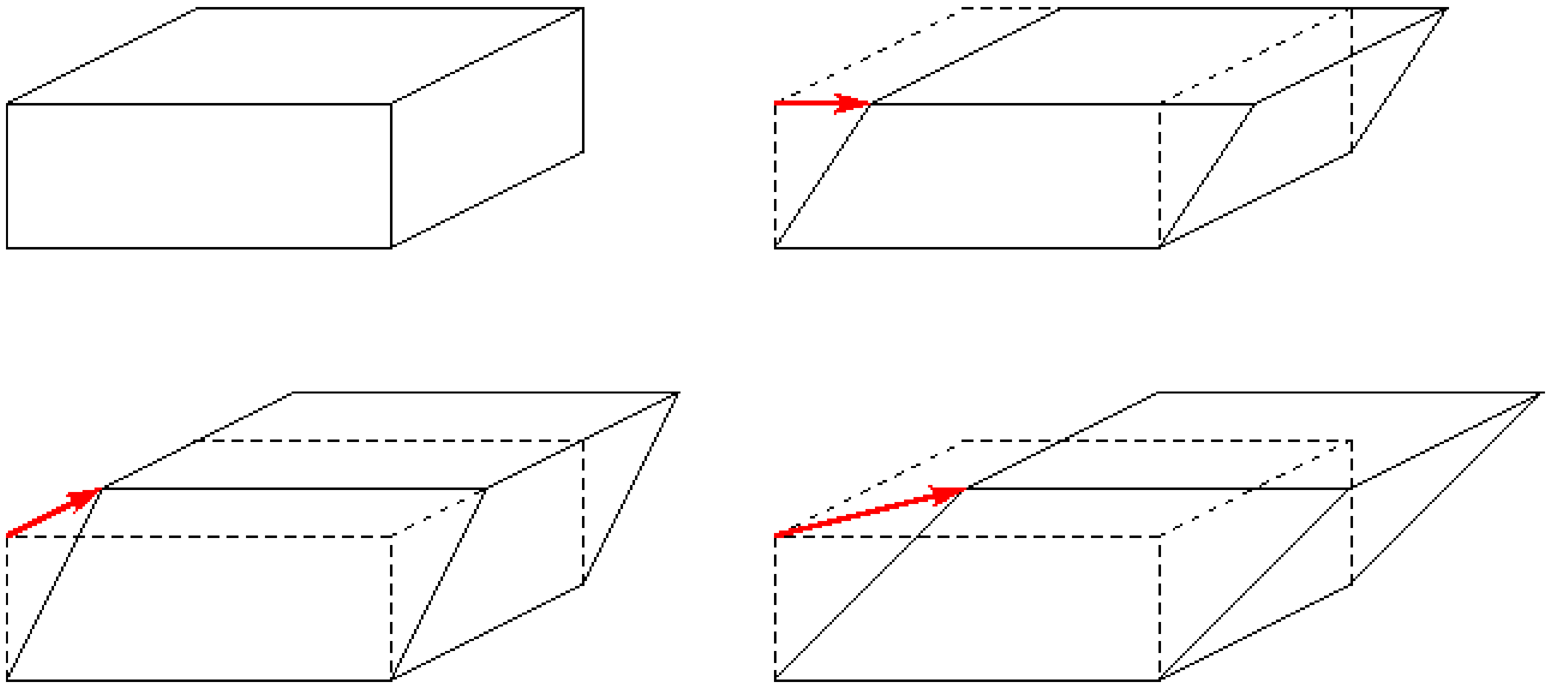




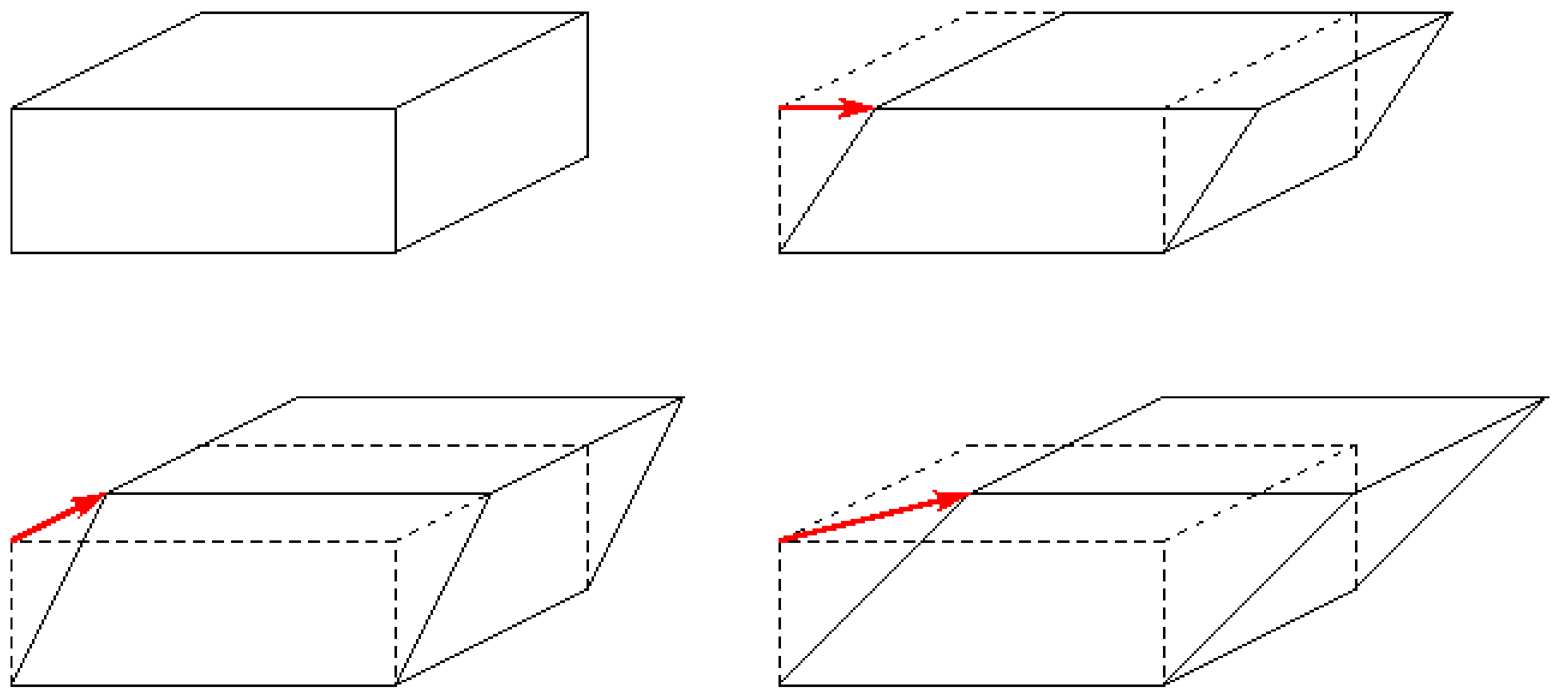
cisaillement permanent :
contrainte oscillante !?

$$\sigma = \frac{\mu}{\sqrt{1-a^2}} \sin\left(\gamma \sqrt{1-a^2}\right)$$

Loi élastique ?



Loi élastique ?



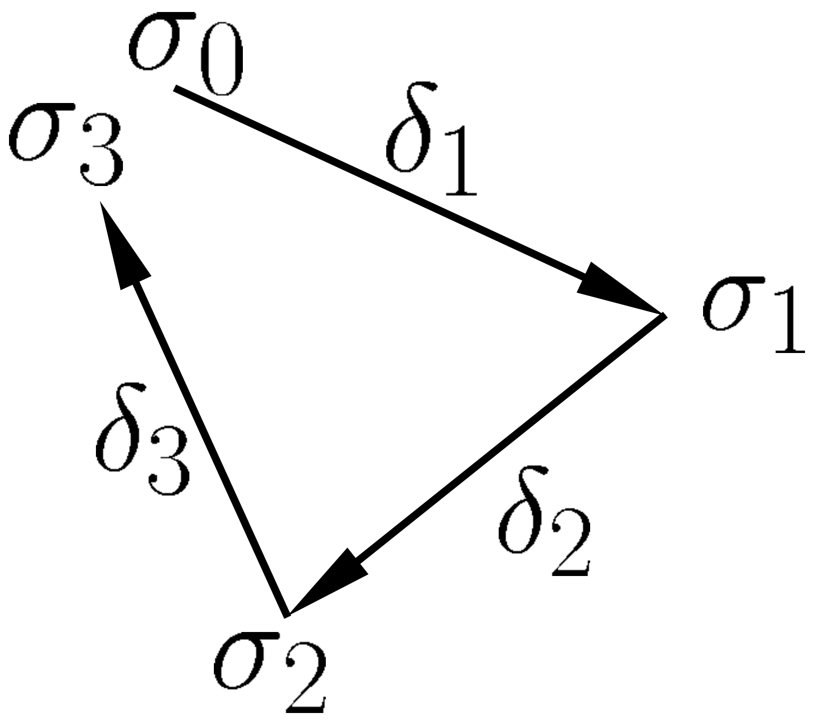
la contrainte finale
dépend du chemin emprunté !?

Kinematic validity of some classical rheological laws

	small elastic deformations	large elastic deformations
upper-convected (a=+1)	valid	valid
lower-convected (a=-1)	valid	valid
corotational (a=0)	valid	not valid
other interpolated (-1<a<+1)	valid	not valid
other derivatives	valid	?

Gordon, Schowalter, 1972

Chemin emprunté



$$\delta_1 + \delta_2 + \delta_3 = \frac{1}{2}(\delta_1\delta_2 - \delta_2\delta_1)$$

(groupes de Lie)

$$\dot{\sigma} = (\Omega\sigma - \sigma\Omega) + f(\sigma) : D$$

Condition :

$$(f'(\sigma) : (f(\sigma) : D_2)) : D_1 - (f'(\sigma) : (f(\sigma) : D_1)) : D_2$$
$$= D_1 D_2 \sigma + \sigma D_2 D_1 - \text{v.v.}$$

Évolutions imaginables

$$\dot{\sigma} = (\Omega\sigma - \sigma\Omega) + f(\sigma) : D$$

$$\mathrm{tr}(D)\ I$$
$$\mathrm{tr}(D)\ \sigma$$
$$\mathrm{tr}(D)\ \sigma^2$$

$$\mathrm{tr}(\sigma D)\ I$$
$$\mathrm{tr}(\sigma D)\ \sigma$$
$$\mathrm{tr}(\sigma D)\ \sigma^2$$

$$\mathrm{tr}(\sigma^2 D)\ I$$
$$\mathrm{tr}(\sigma^2 D)\ \sigma$$
$$\mathrm{tr}(\sigma^2 D)\ \sigma^2$$

$$D$$
$$\sigma D + D\sigma$$
$$\sigma^2 D + D\sigma^2$$
$$\sigma D\sigma$$
$$\sigma^2 D\sigma + \sigma D\sigma^2$$
$$\sigma^2 D\sigma^2$$

Évolutions imaginables

$$\dot{\sigma} = (\Omega\sigma - \sigma\Omega) + f(\sigma) : D$$

$\mathrm{tr}(D)\ I$ ★

$\mathrm{tr}(D)\ \sigma$ ★

$\mathrm{tr}(D)\ \sigma^2$ ★

$\mathrm{tr}(\sigma D)\ I$

$\mathrm{tr}(\sigma D)\ \sigma$ ★

$\mathrm{tr}(\sigma D)\ \sigma^2$

$\mathrm{tr}(\sigma^2 D)\ I$

$\mathrm{tr}(\sigma^2 D)\ \sigma$ ★

$\mathrm{tr}(\sigma^2 D)\ \sigma^2$

D

$\sigma D + D\sigma$

$\sigma^2 D + D\sigma^2$

$\sigma D\sigma$

$\sigma^2 D\sigma + \sigma D\sigma^2$

$\sigma^2 D\sigma^2$

Évolutions imaginables

$$\dot{\sigma} = (\Omega\sigma - \sigma\Omega) + f(\sigma) : D$$

$\mathrm{tr}(D)\ I$ ★

$\mathrm{tr}(D)\ \sigma$ ★

$\mathrm{tr}(D)\ \sigma^2$ ★

D ★

$\mathrm{tr}(\sigma D)\ I$

$\mathrm{tr}(\sigma D)\ \sigma$ ★

$\sigma D + D\sigma$ ★

$\mathrm{tr}(\sigma D)\ \sigma^2$

$\sigma^2 D + D\sigma^2$

$\sigma D\sigma$

$\mathrm{tr}(\sigma^2 D)\ I$

$\mathrm{tr}(\sigma^2 D)\ \sigma$ ★

$\sigma^2 D\sigma + \sigma D\sigma^2$

$\mathrm{tr}(\sigma^2 D)\ \sigma^2$

$\sigma^2 D\sigma^2$

Évolutions admissibles

$$\dot{\sigma} = (\Omega\sigma - \sigma\Omega) \pm (\sigma D + D\sigma)$$

$$+2\,\mu\,D$$

$$+\mathrm{tr}(D)\,\left(s_1(\sigma) I + s_2(\sigma)\sigma + s_3(\sigma)\,\sigma^2\right)$$

$$+s_5(\sigma)\,\mathrm{tr}(\sigma D)\,\sigma$$

$$+s_8(\sigma)\,\mathrm{tr}(\sigma^2 D)\,\sigma$$

$$+\cdots ?$$

signification mécanique ?

Évolutions admissibles

rotation

sur-convecté / sous-convecté

$$\dot{\sigma} = (\Omega\sigma - \sigma\Omega) \pm (\sigma D + D\sigma)$$

$$+2\,\mu\,D$$ élasticité linéaire

compressibilité

$$+\mathrm{tr}(D)\,\left(s_1(\sigma)I + s_2(\sigma)\sigma + s_3(\sigma)\,\sigma^2\right)$$

$$+s_5(\sigma)\,\mathrm{tr}(\sigma D)\,\sigma$$

effets non linéaires

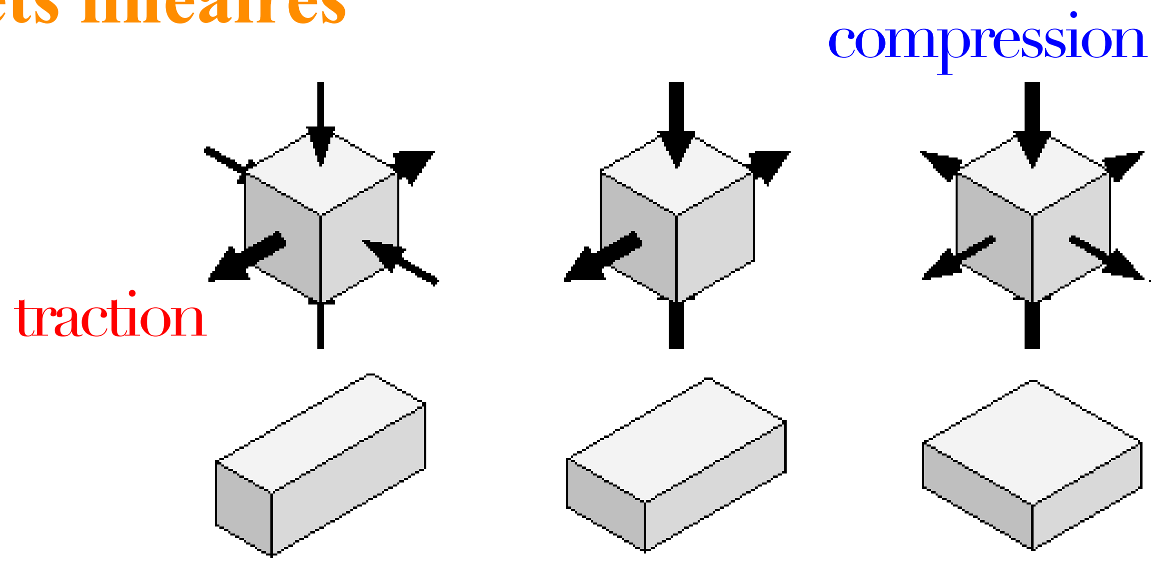
$$+s_8(\sigma)\,\mathrm{tr}(\sigma^2 D)\,\sigma$$

$$+\cdots ?$$

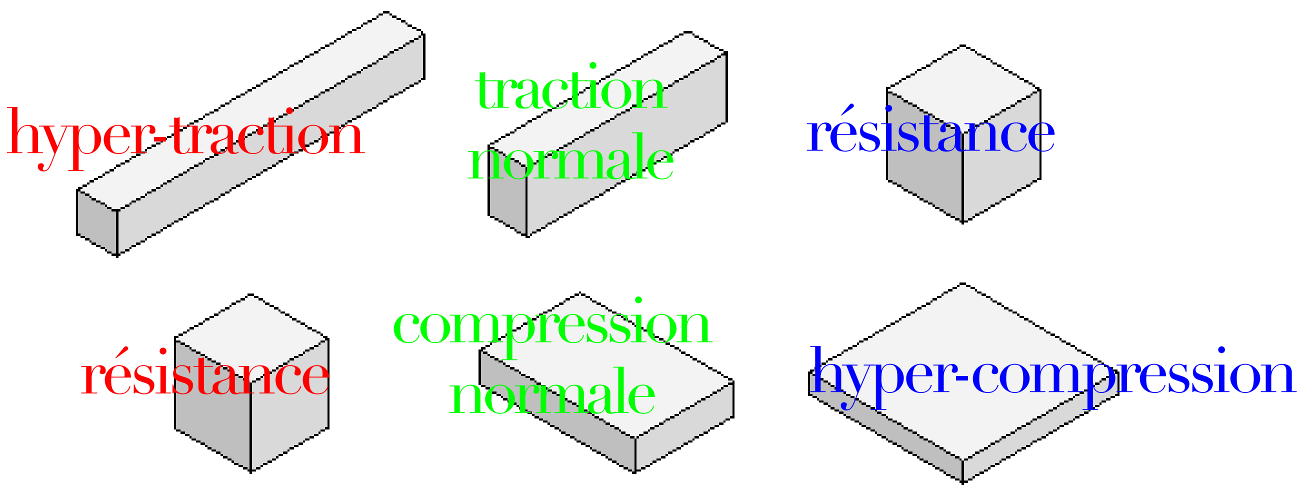
Valable pour les mousses

et pour tout système très déformable élastiquement !

Effets linéaires



Effets non linéaires



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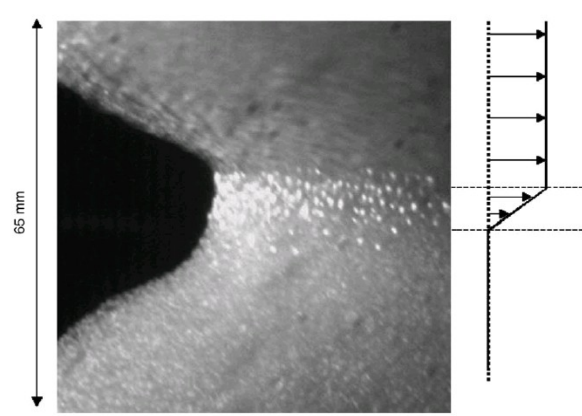
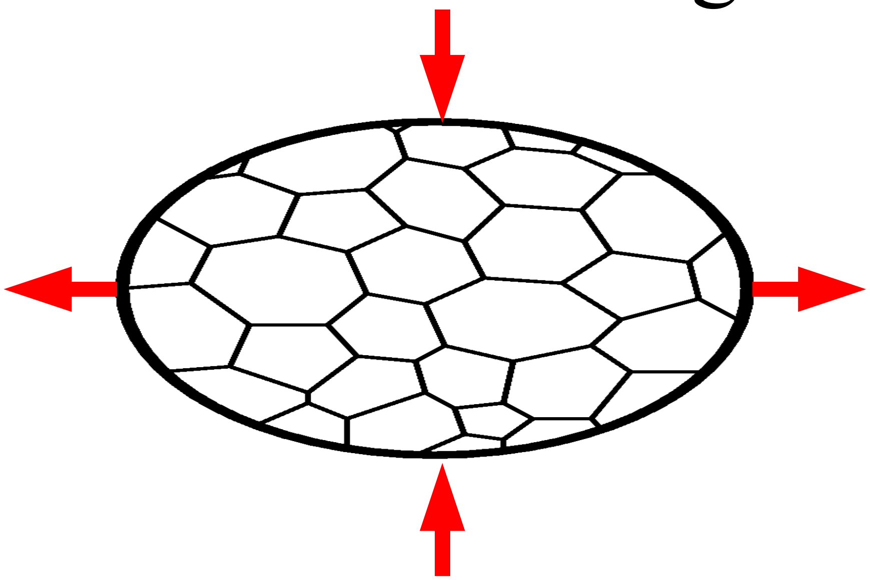


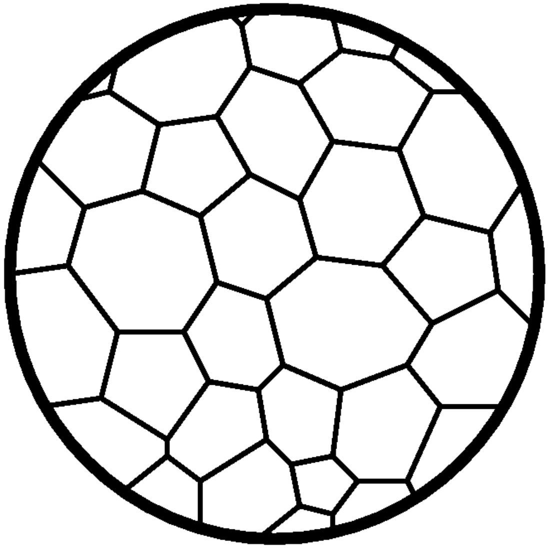
Modules élastiques

$$R \simeq 1\,\mathrm{mm}$$

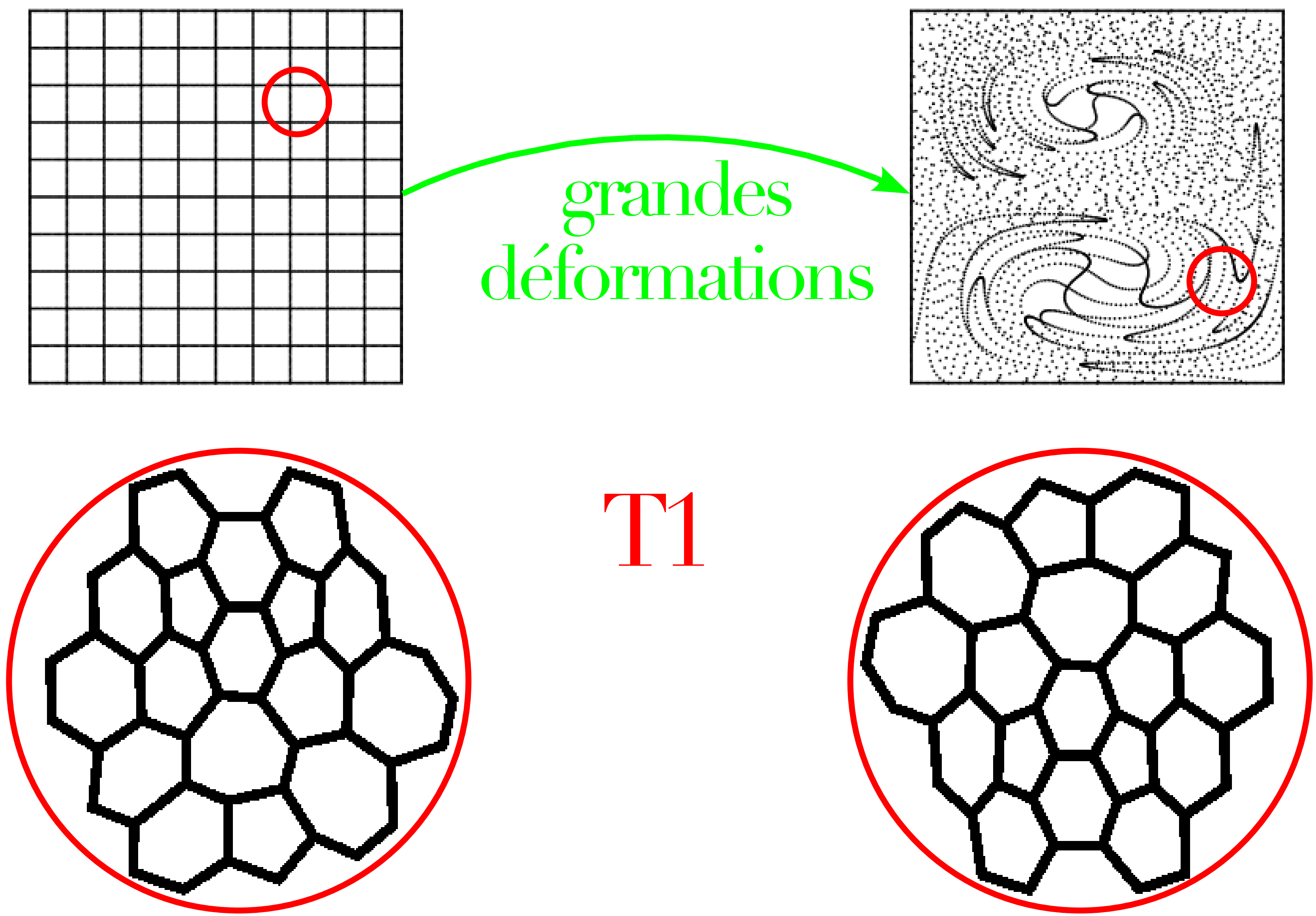
Cisaillement / élongation



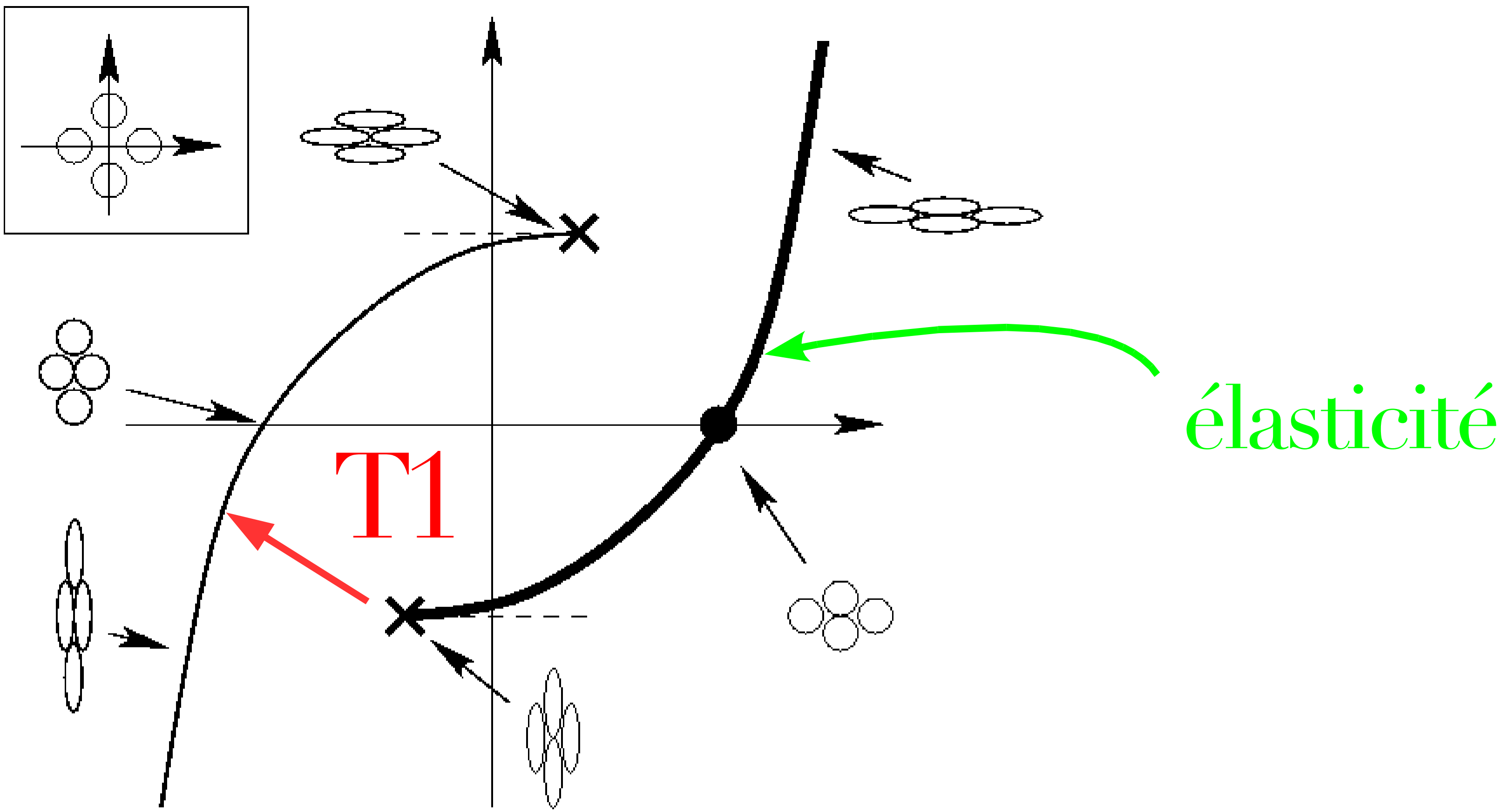
$$\mu \sim \gamma/R \simeq 10 - 100\,\mathrm{Pa}$$



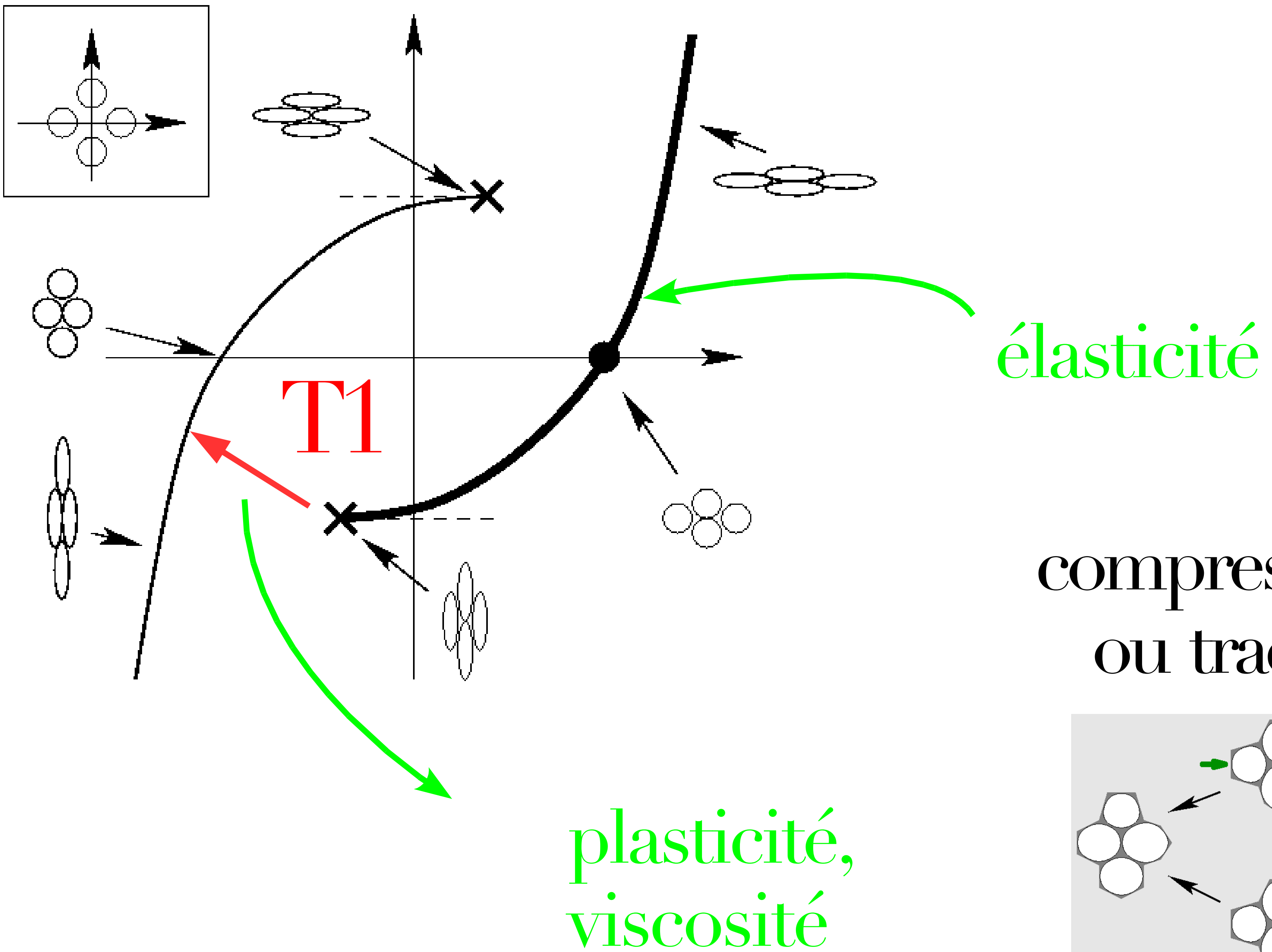
Origine de la plasticité d'une mousse



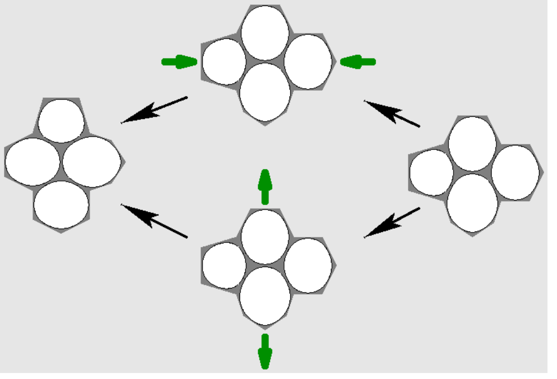
Origine de la plasticité d'une mousse



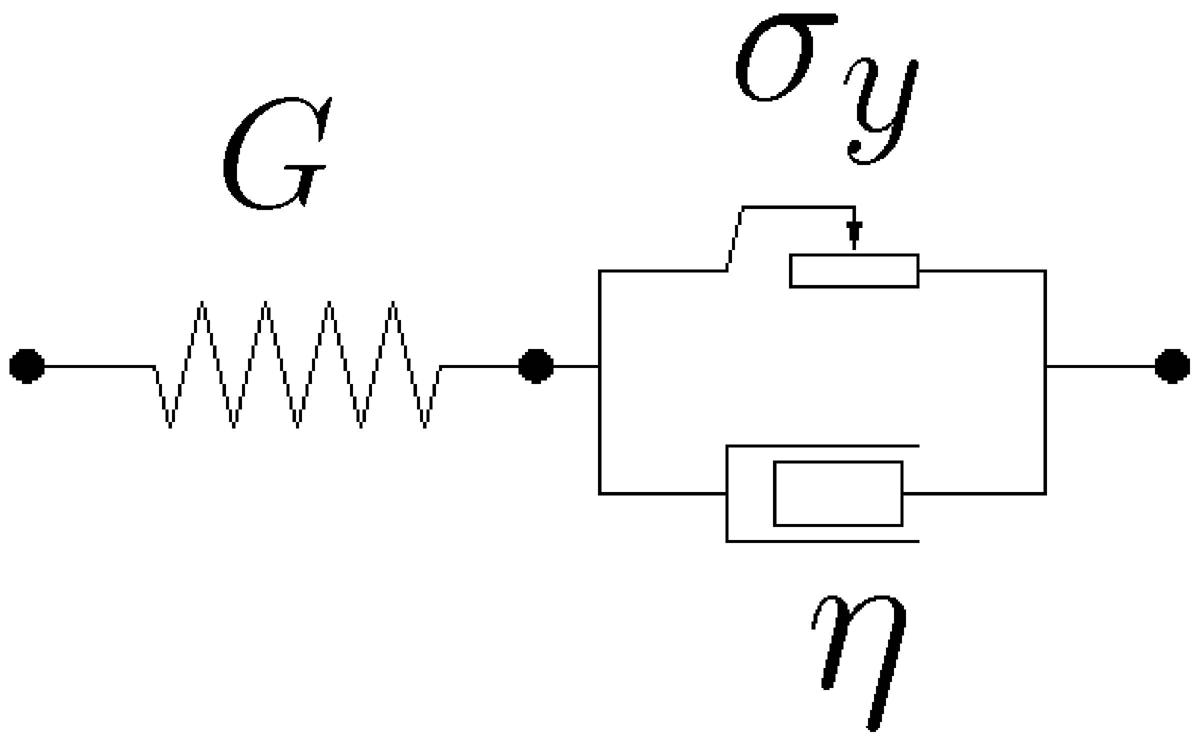
Origine de la plasticité d'une mousse



compression
ou traction



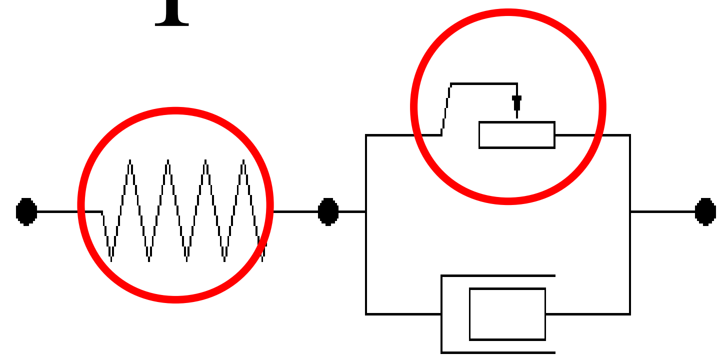
Comportement simplifié



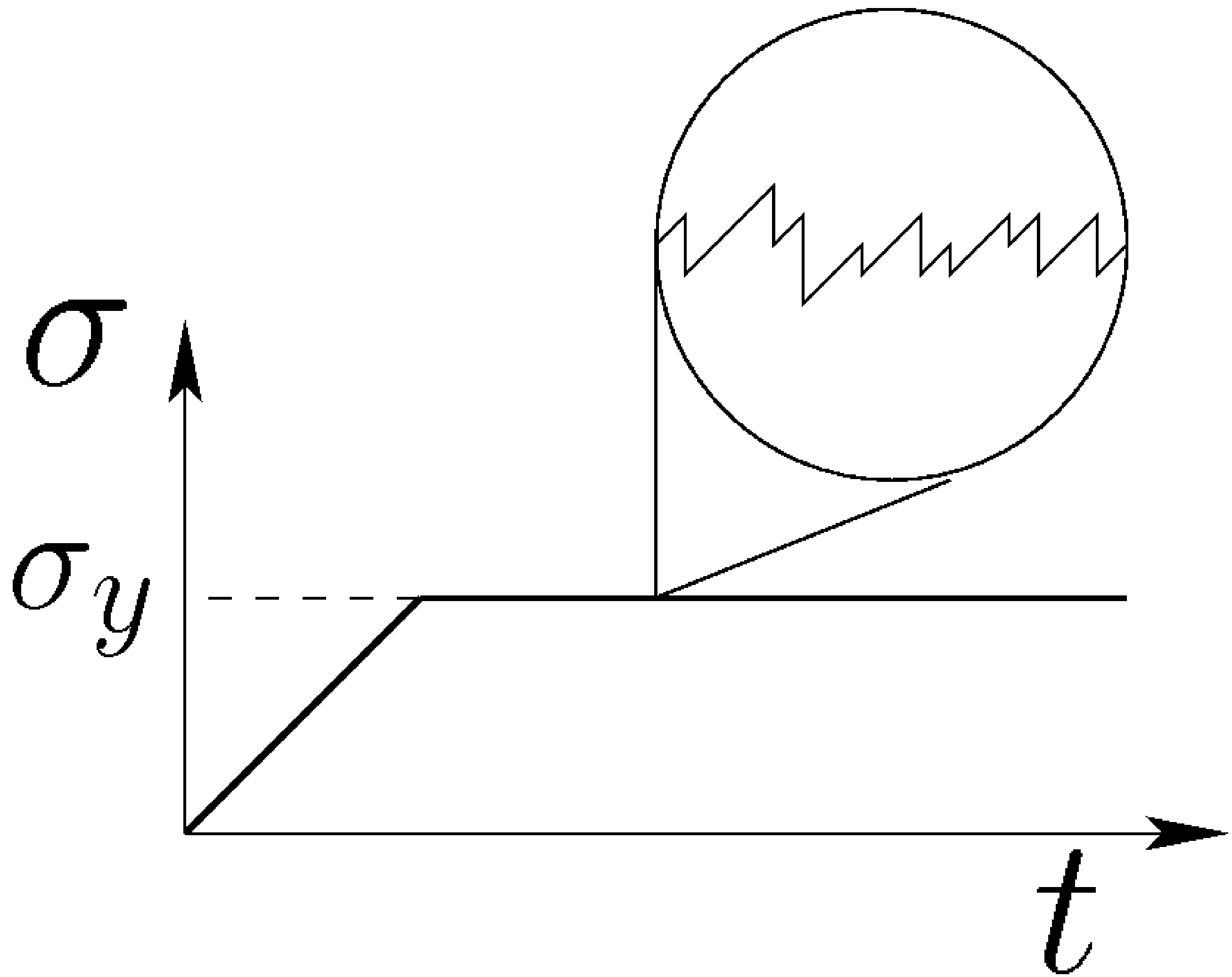
Bingham

Comportement simplifié

Cisaillement quasistatique

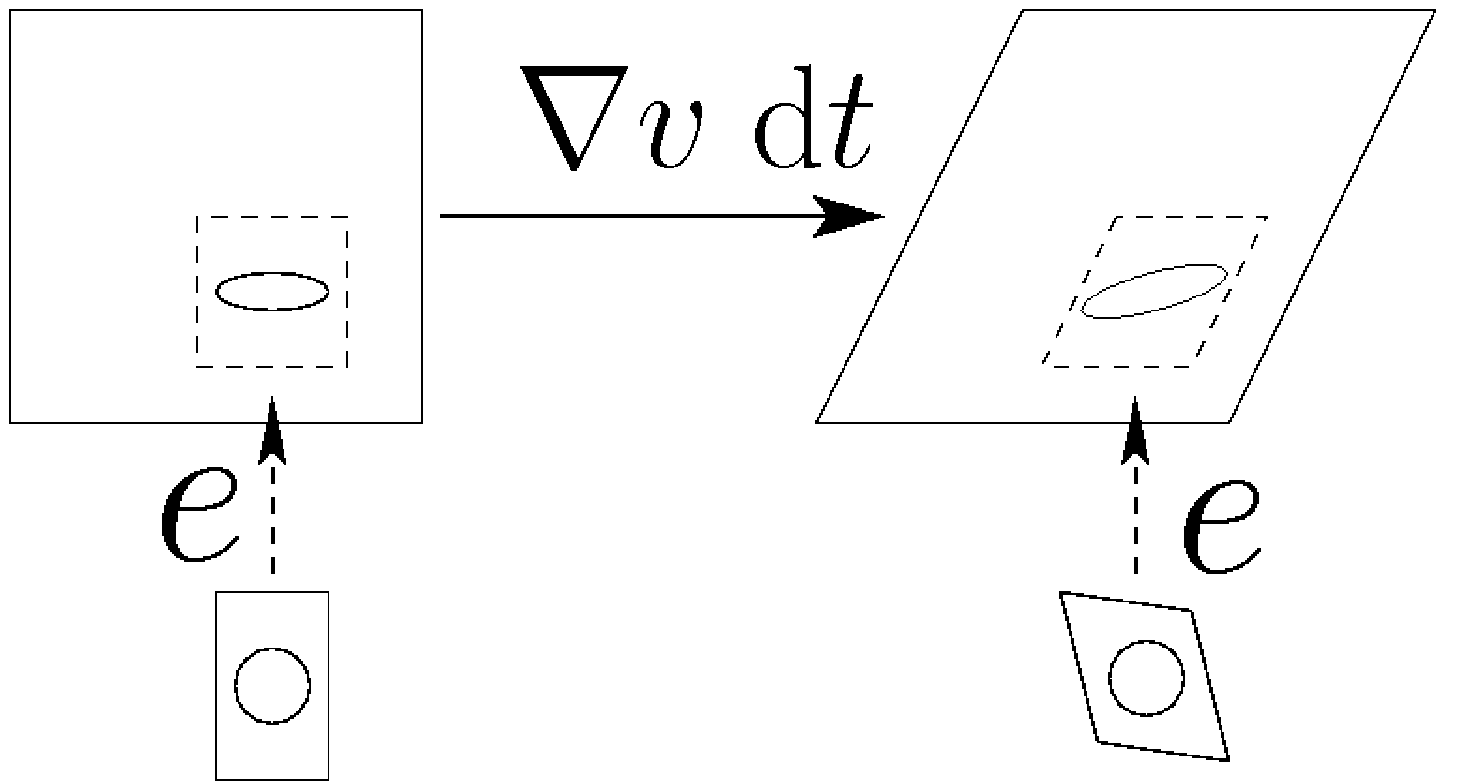


Bingham



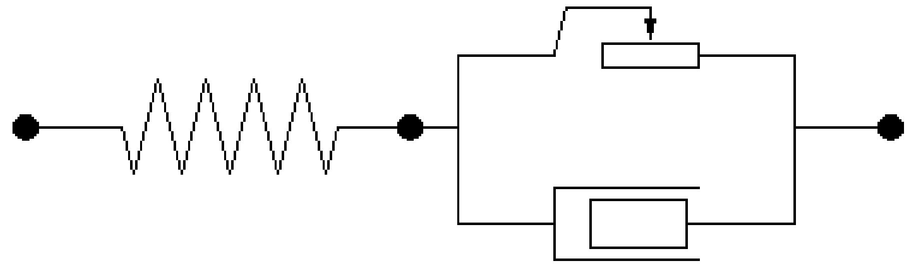
Formulation tensorielle

$$\frac{\mathrm{D}e}{\mathrm{D}t} = f\left(\nabla v,\ e\right)$$



$$\frac{\mathrm{D}e}{\mathrm{D}t} = \frac{\partial e}{\partial t} + (v\cdot\nabla)e - \nabla v\cdot e - e\cdot\nabla v^{\mathrm{T}}$$

Formulation tensorielle



Déformations élastique et plastique

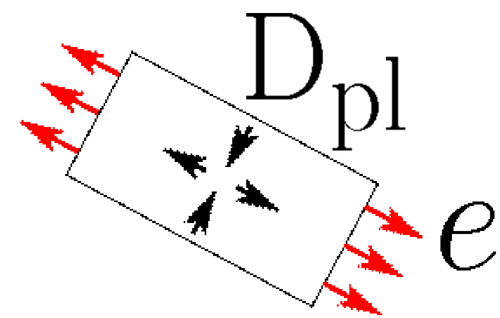
$$\mathrm{D} = \frac{\mathrm{D}e}{\mathrm{D}t} + \mathrm{D}_{\mathrm{pl}}$$

$$\mathrm{D}_{\mathrm{pl}} = \mathrm{D}_{\mathrm{pl}}(s) = \mathrm{D}_{\mathrm{pl}}(e)$$

Déformation plastique

Forme tensorielle générale

$$\mathrm{D}_{\mathrm{pl}} = A_0\,\mathrm{I} + A_1\,e + A_2\,e^2$$



Incompressibilité

Deux fonctions indépendantes

Dissipation positive

Second principe

Système complet d'équations

$$e \leftrightarrow s$$

$$\frac{\partial e}{\partial t} + (v \cdot \nabla) e = D + \nabla v \cdot e + e \cdot \nabla v^{\mathrm{T}}$$
$$- A_0(e)\,\mathrm{I} - A_1(e)\,e - A_2(e)\,e^2$$

$$\mathrm{div}\, s = \mathrm{grad}\, p$$

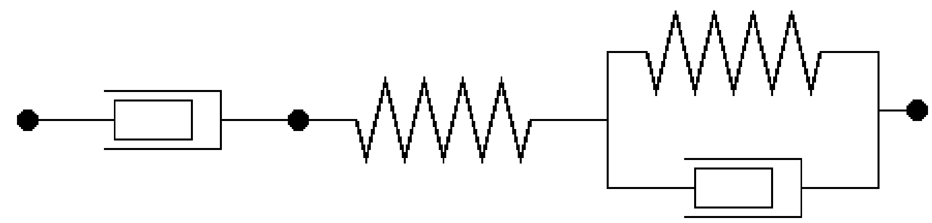
$$\mathrm{div}\, v = 0$$

Bénito et al. Eur. Phys. J. E 2008

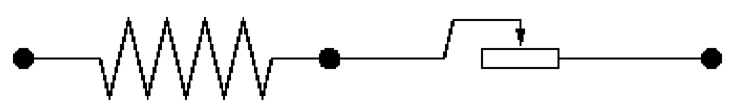
Autres modèles :
modèles scalaires

Höhler, Cohen-Addad (2004)

fluage à faible contrainte
(mûrissement,...)



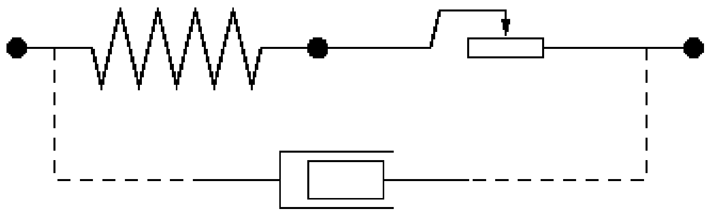
Janiaud, Weaire, Hutzler (2006)



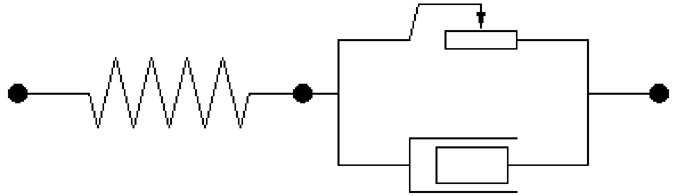
mousse 2D entre deux plaques,
vitesse radiale $v_\theta(r)$

Autres modèles : modèles tensoriels

Marmottant, Graner (2007) linéaire

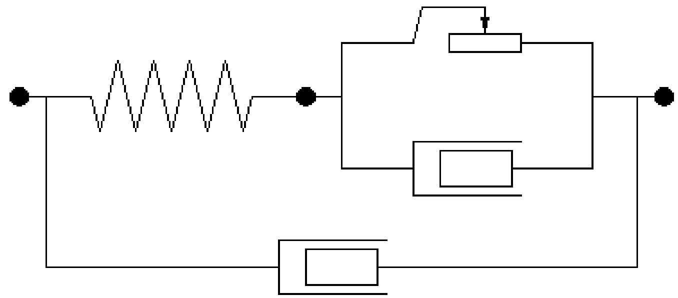


Takeshi, Sekimoto (2005) $v_z(x,\ y)$ linéaire

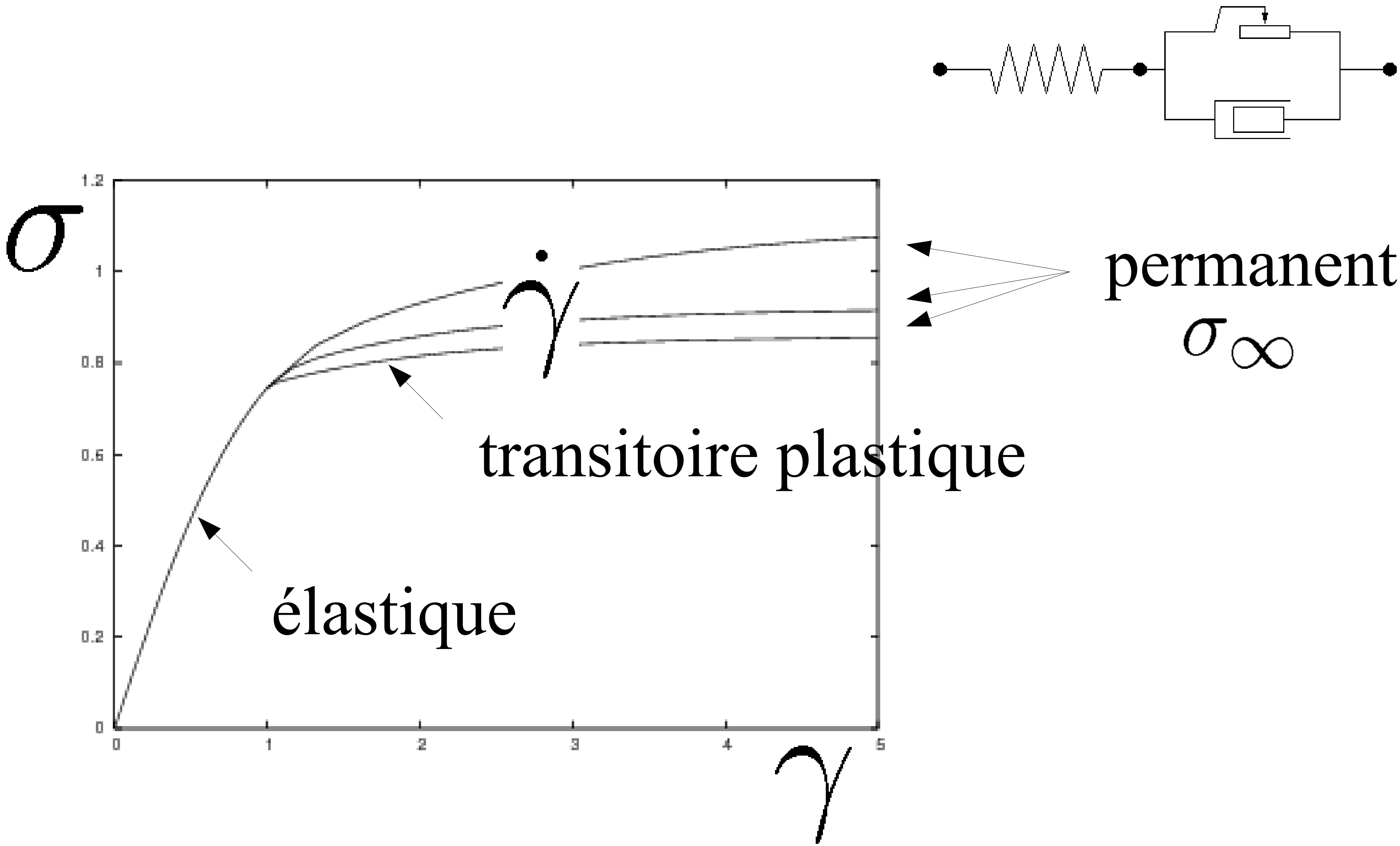


Bénito et al. (2008) non-linéaire (générique)

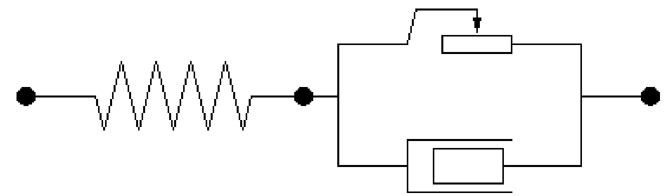
Saramito (2007) linéaire



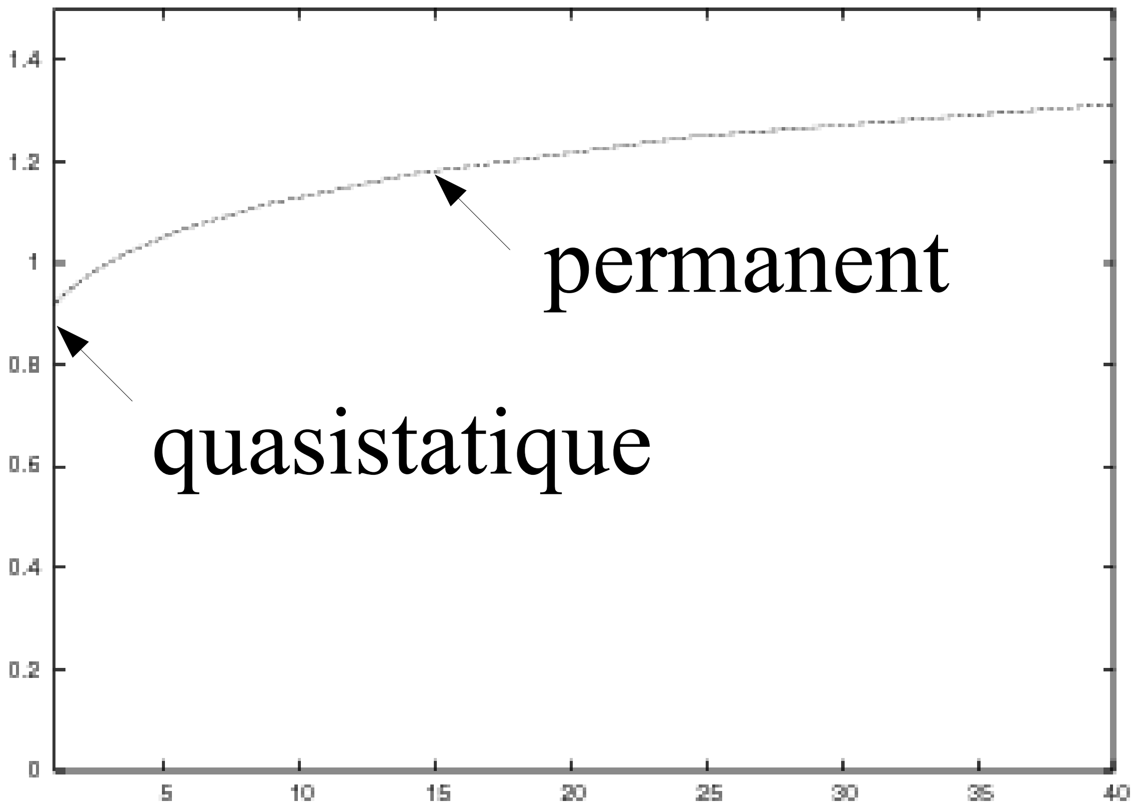
Cisaillement homogène constant



Cisaillement homogène constant

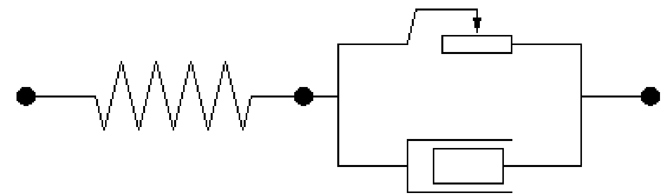


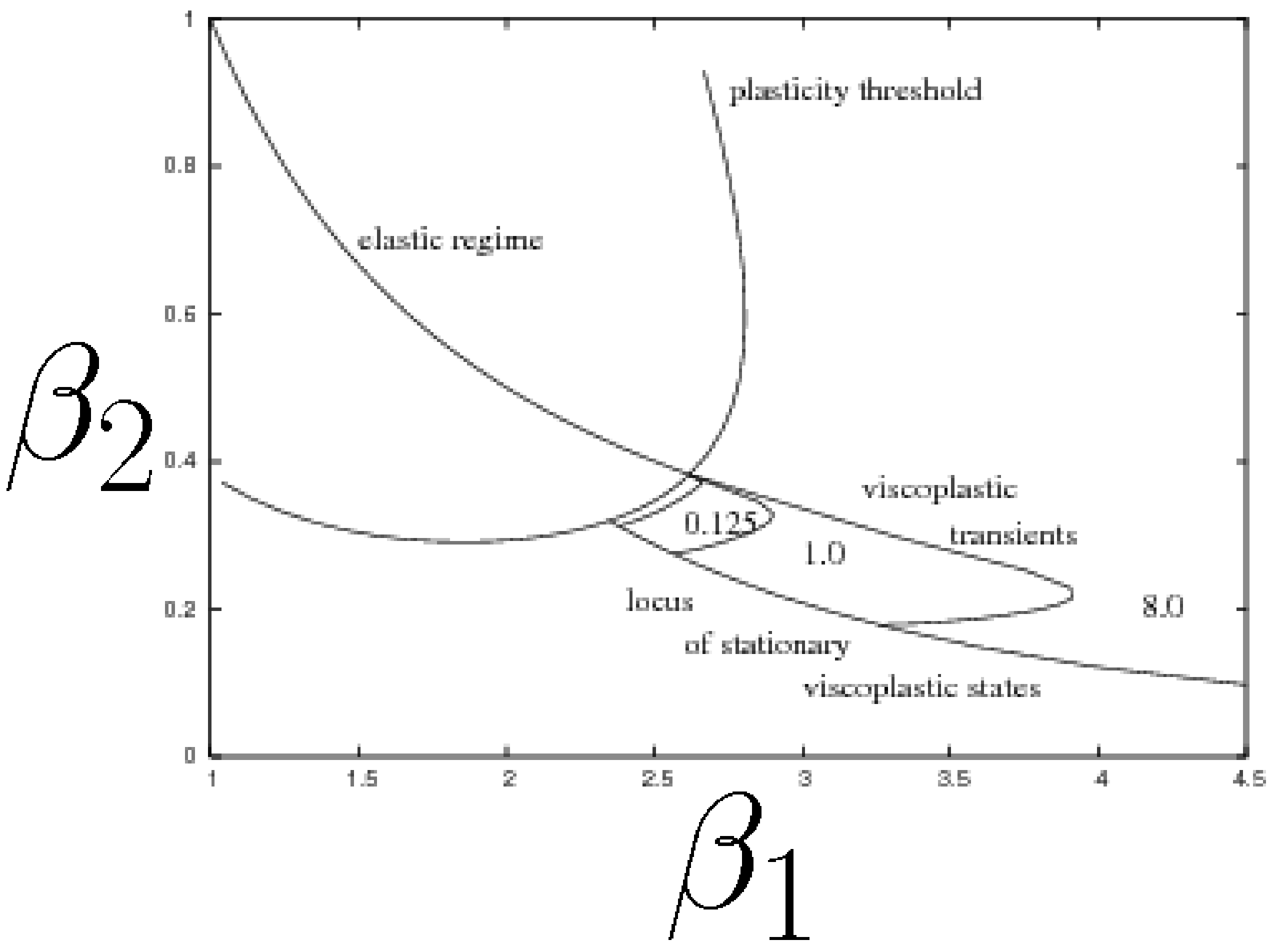
σ_∞

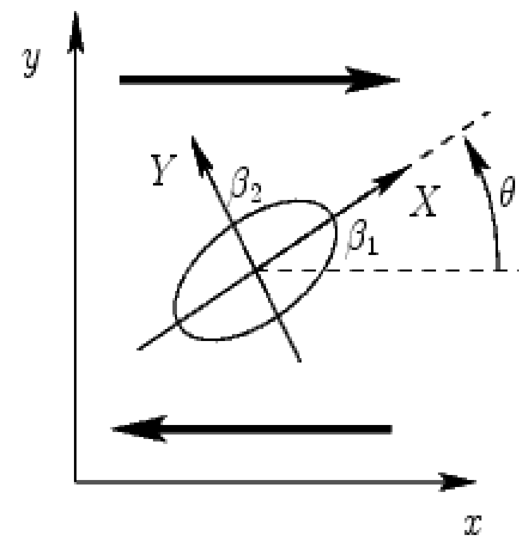


$\dot{\gamma}$

Cisaillement homogène constant







$$\beta_1 \, \beta_2 \, \beta_3 = 1$$

Plan

Mémoire du matériau et système de coordonnées

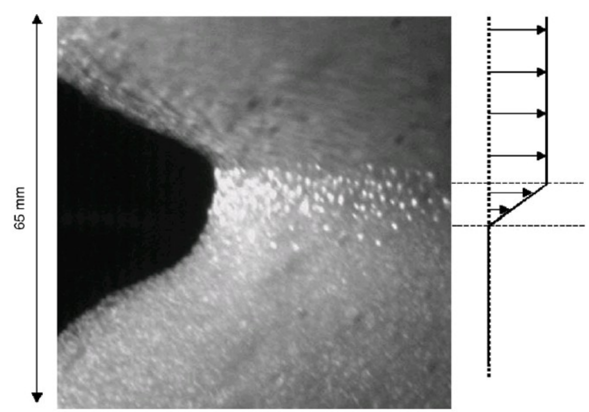
Élasticité :

définitions de la déformation

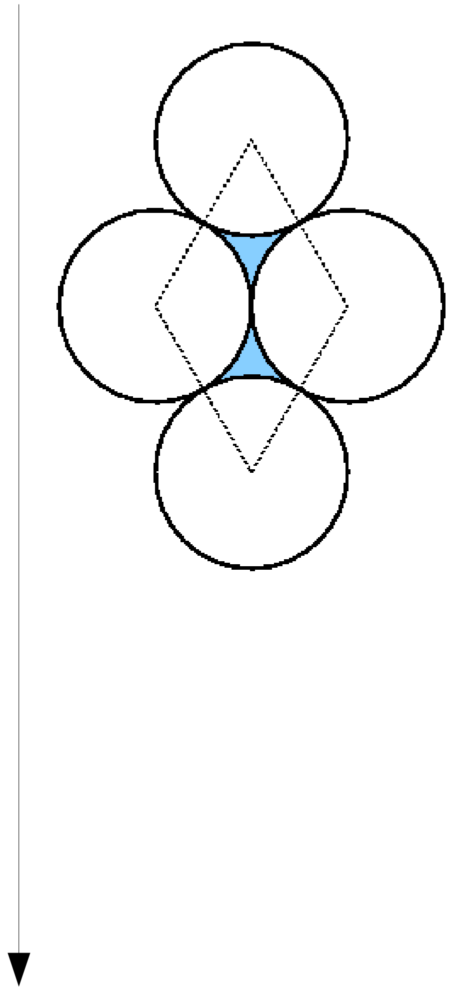
lois admissibles d'évolution de la contrainte

Plasticité, viscosité, modèle complet

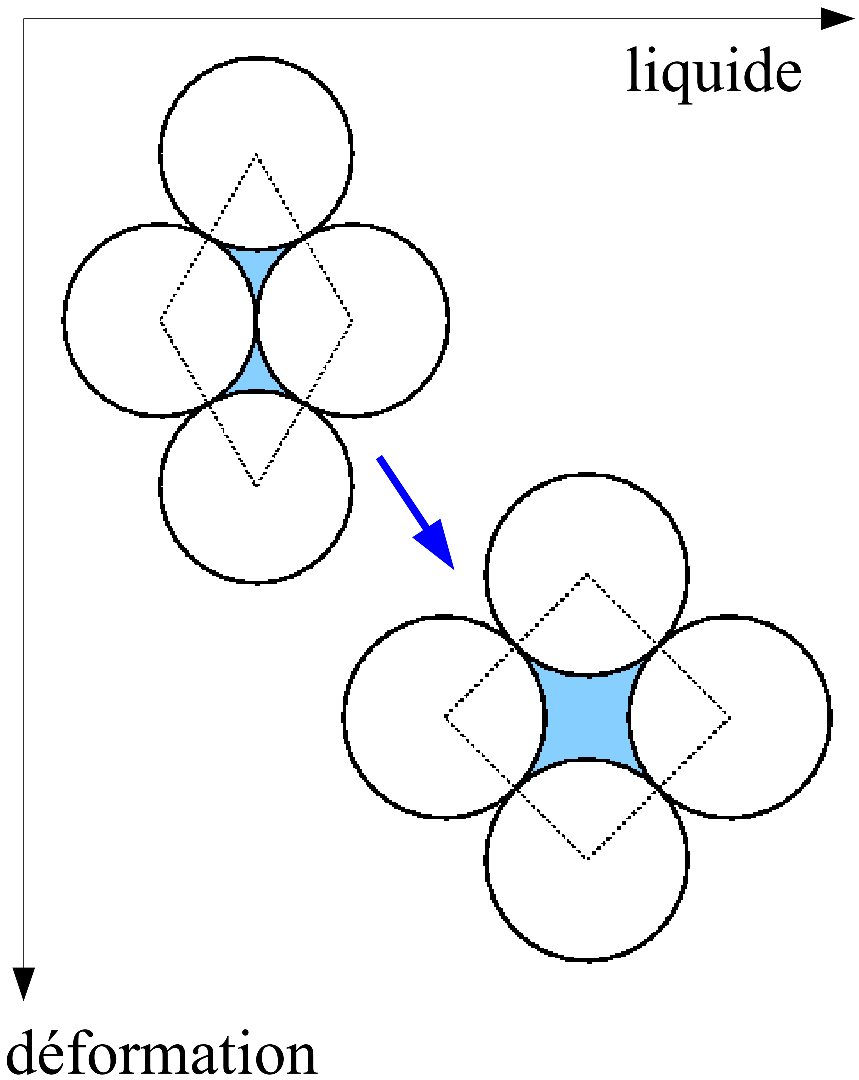
Dilatance



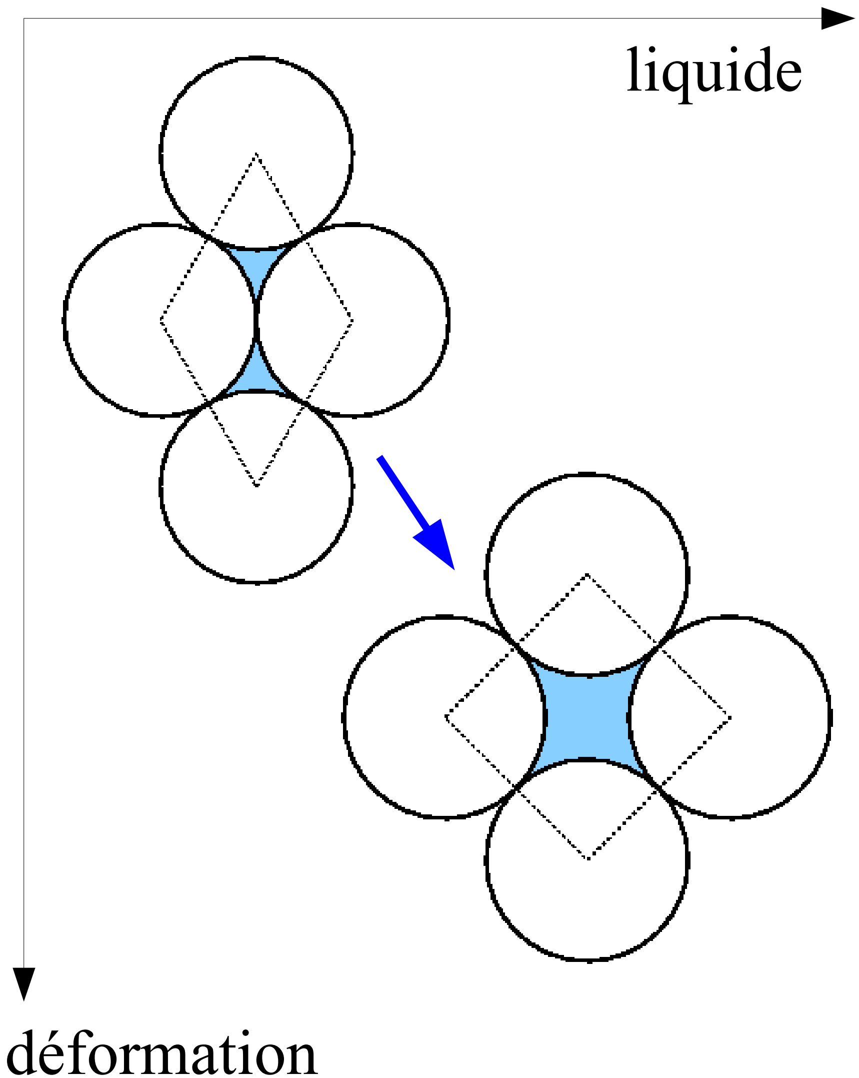
Dilatance

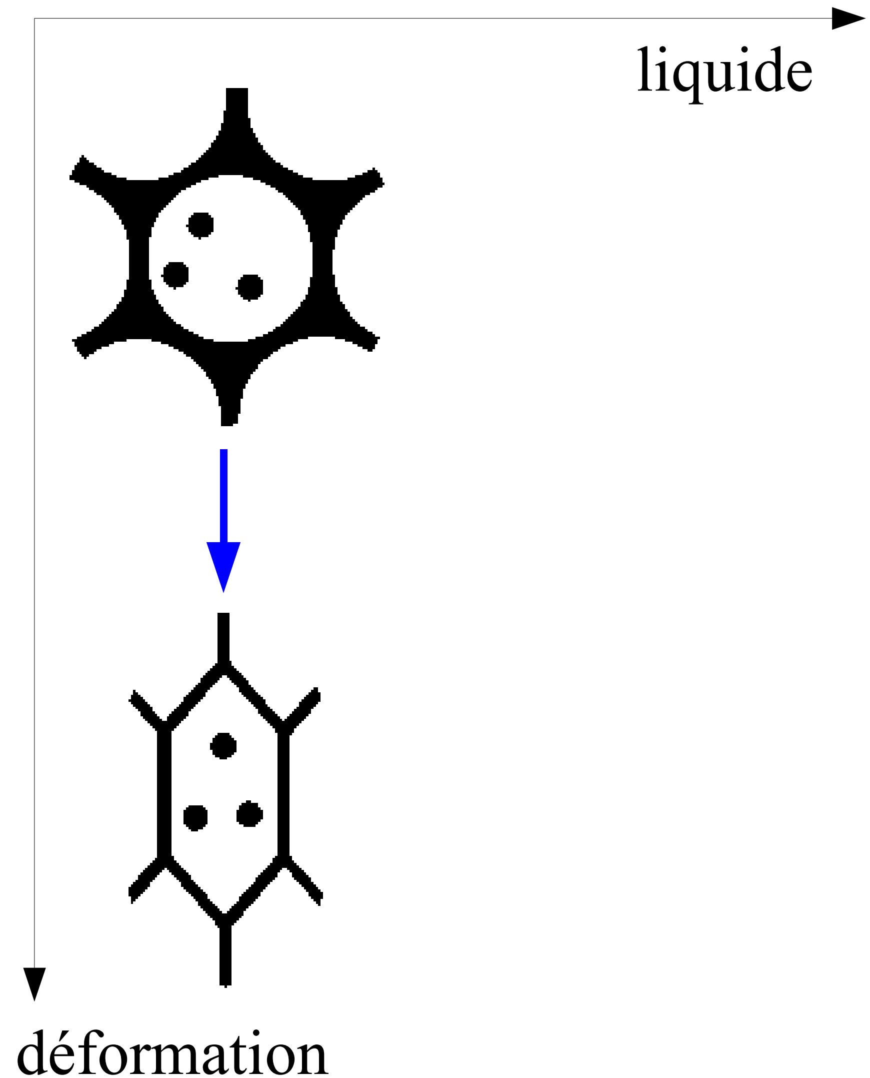


Dilatance

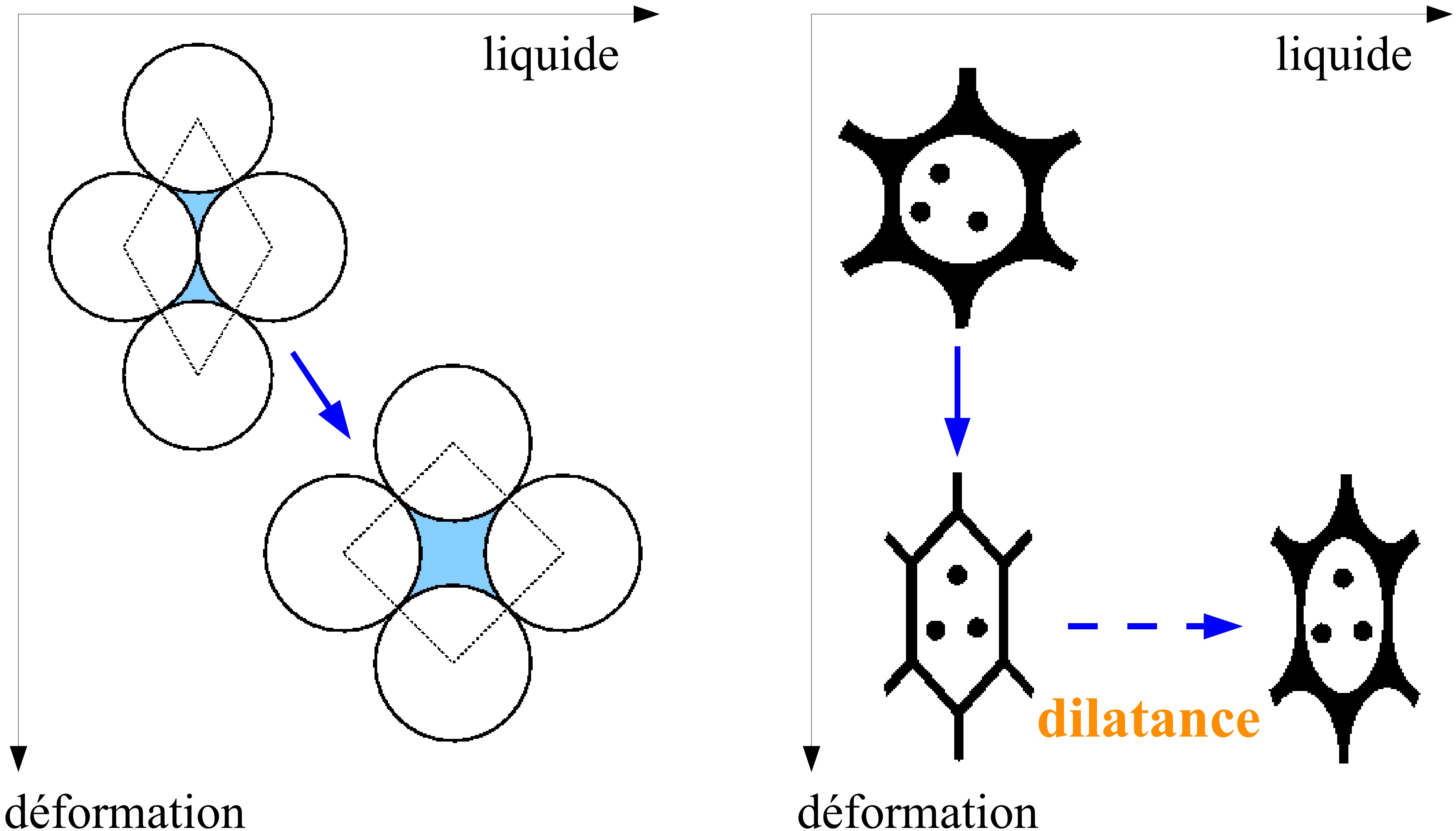


Dilatance

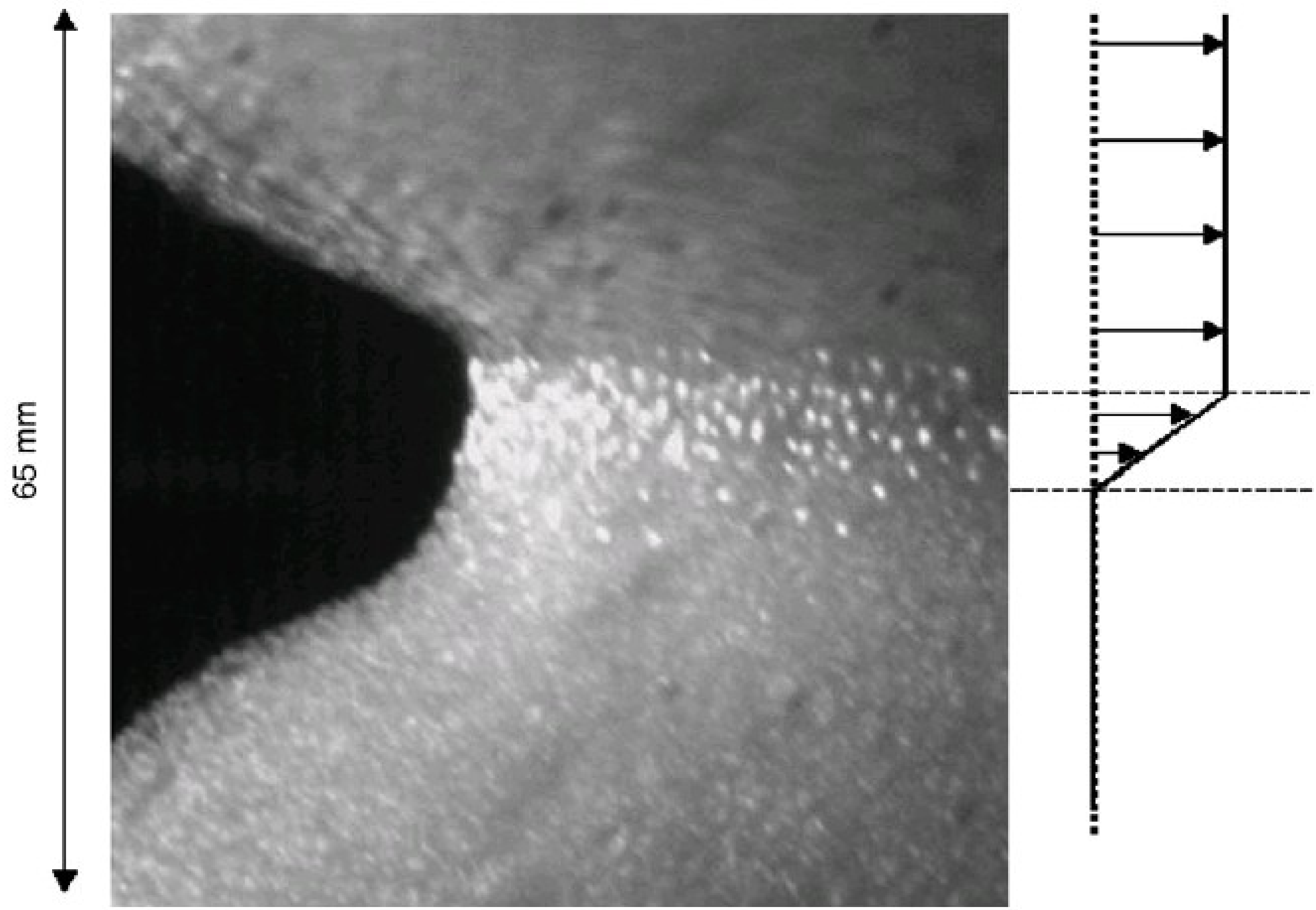




Dilatance



Dilatance



S. Marze, A. Saint-Jalmes, D. Langevin, 2005

pression de liquide
plus basse ?

volume de liquide
plus grand ?

importance :
bandes de cisaillement,...

Dilatance statique

D. Weaire, S. Hutzler, F. Rioual, 2003, 2005

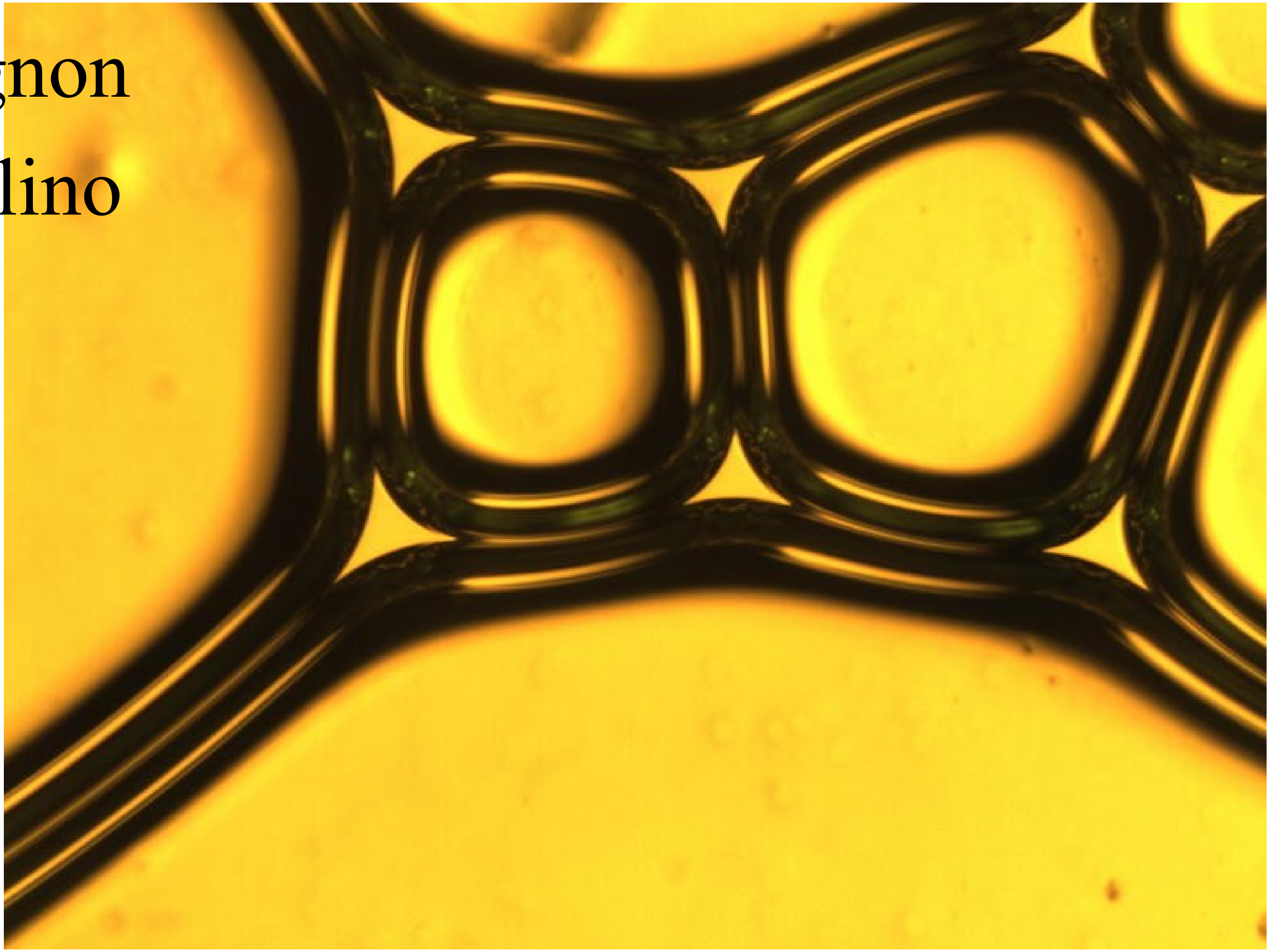
Dilatance dans une mousse 2D

(explication thermodynamique)

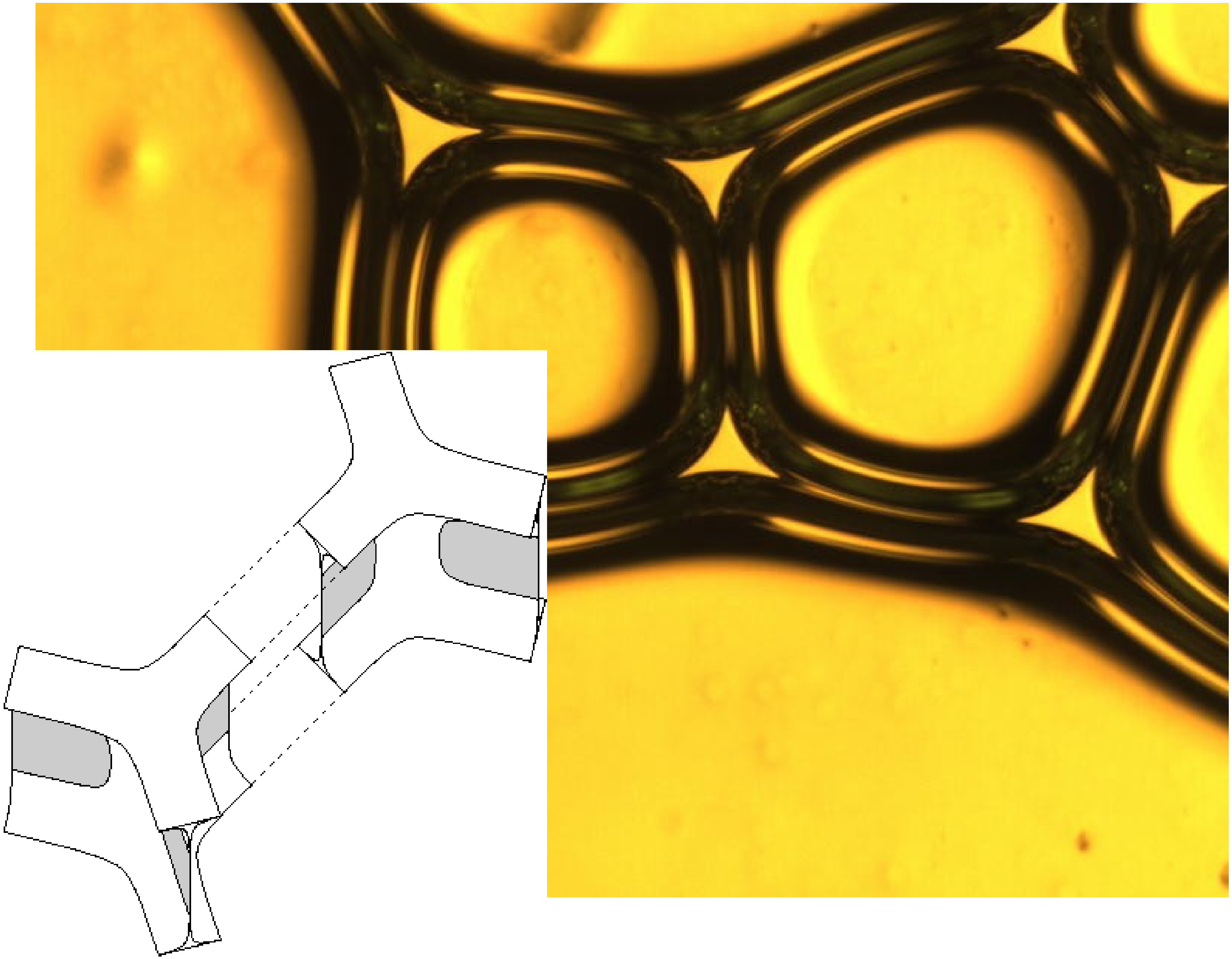
Dilatance = géométrie ?

P. Rognon

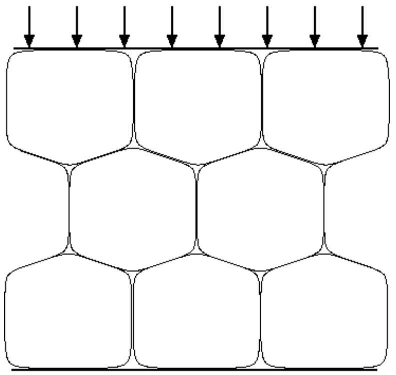
F. Molino

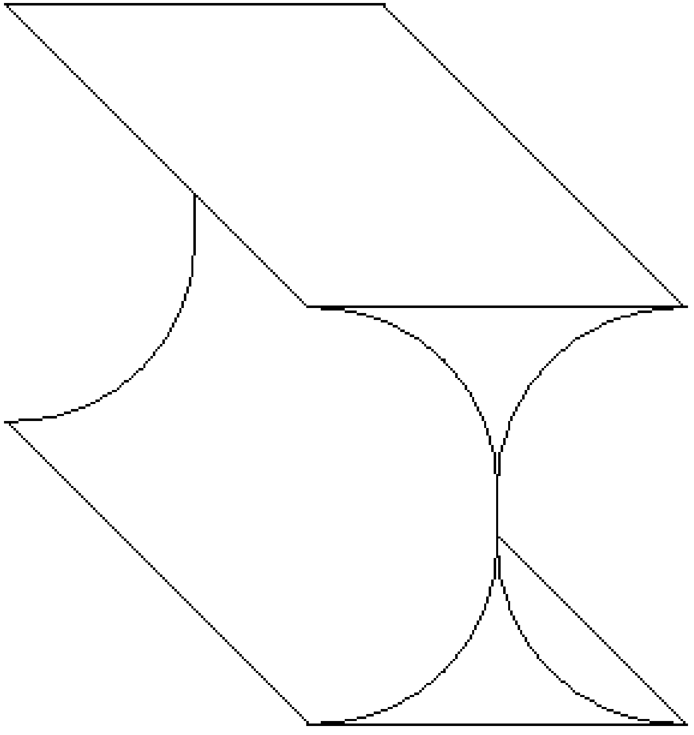


Dilatance = géométrie ?

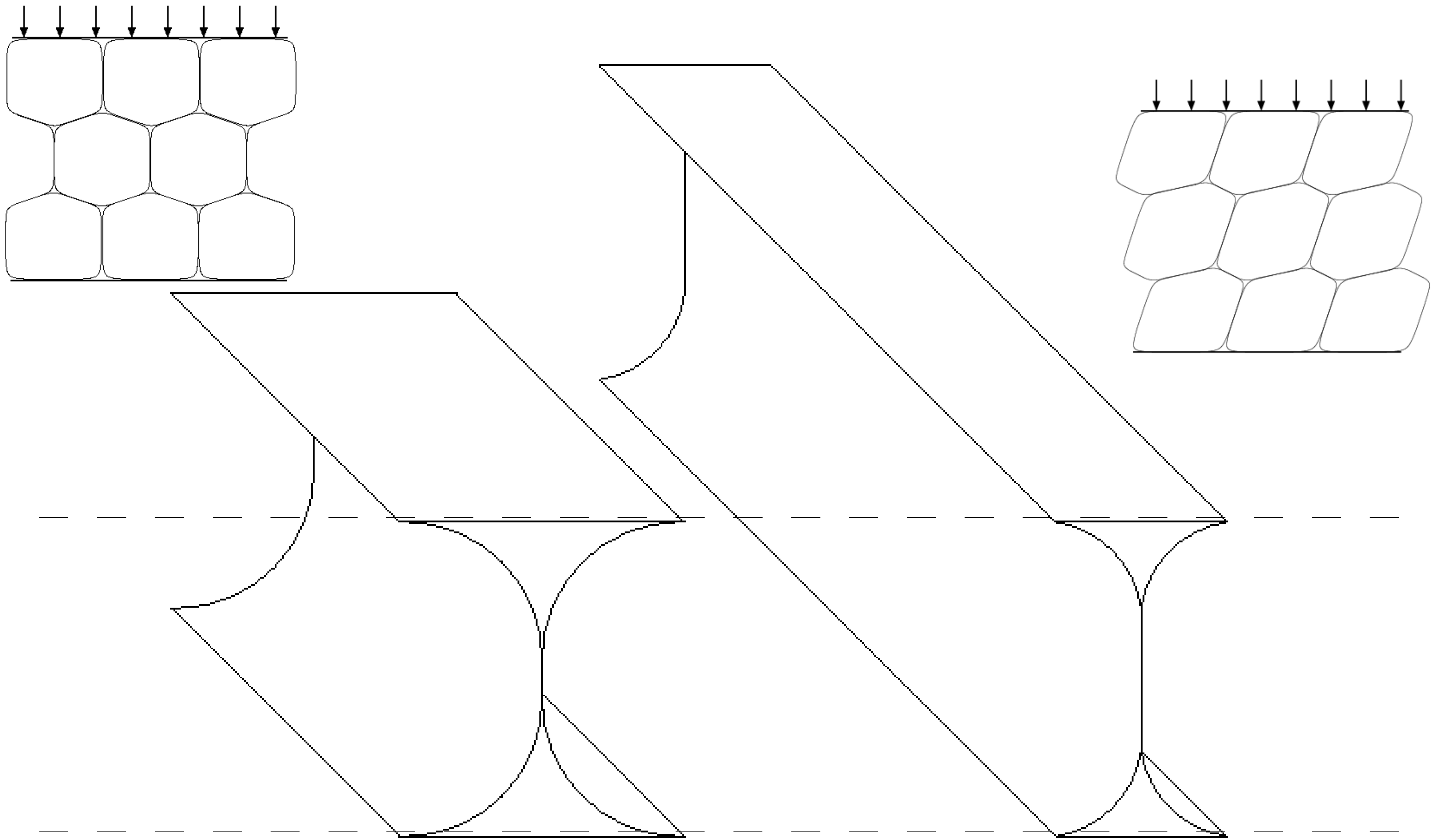


Dilatance = géométrie ?





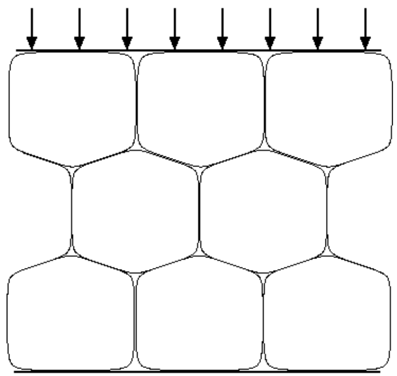
Dilatance = géométrie ?

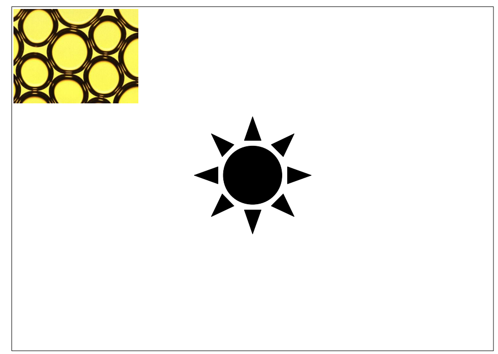


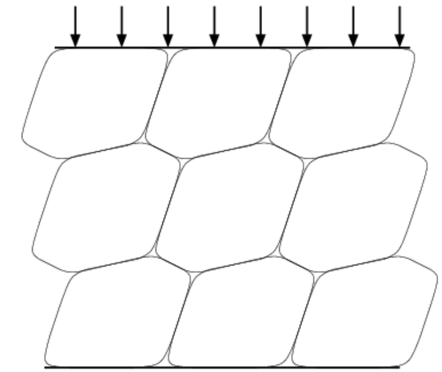
volume = périmètre x section

Test expériental

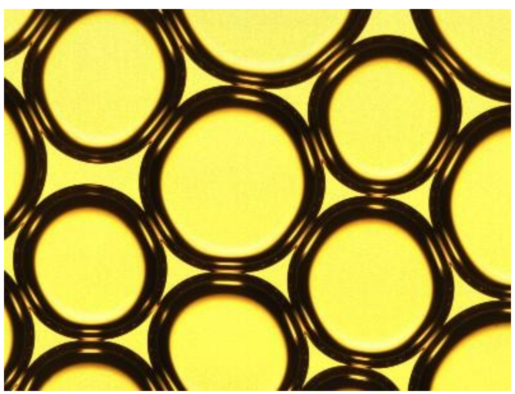
C. Derec, F. Élias, M.-A. Guedeau-Boudeville, D. Osmani

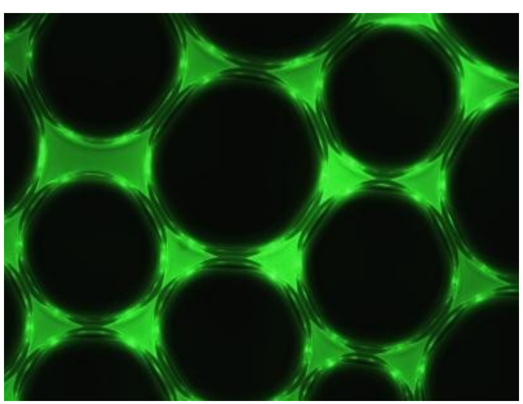




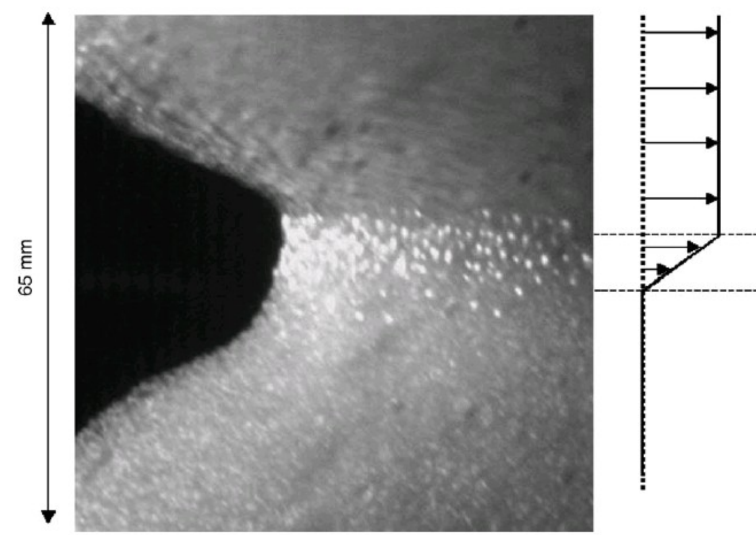


Effet attendu : 1,15



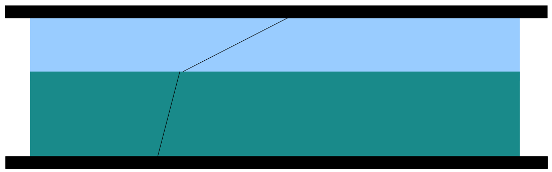


Mesurer la fraction liquide



S. Marze, A. Saint-Jalmes, D. Langevin, 2005

Bandes de cisaillement



visqueux

$$\eta(\dot\gamma)$$

$$\eta(\varphi)$$

$$\dot\varphi = \ldots$$

viscoélastique

seuil plastique → **propagation**

$$\dot\sigma(\dot\gamma,\ \sigma)$$

$$\dot\sigma(\dot\gamma,\ \sigma,\ \varphi)$$ → **désordre**

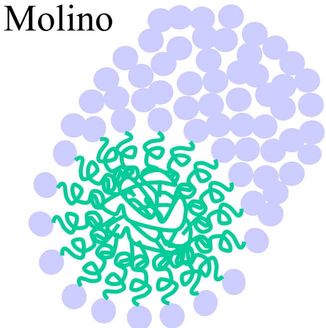
$$\dot\varphi = \ldots$$ → **simulation**

dilatance → **élasticité**

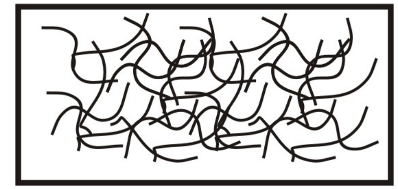
Cates, Ajdari, Aradian, Olmsted, Lu

Micelles géantes

F. Molino



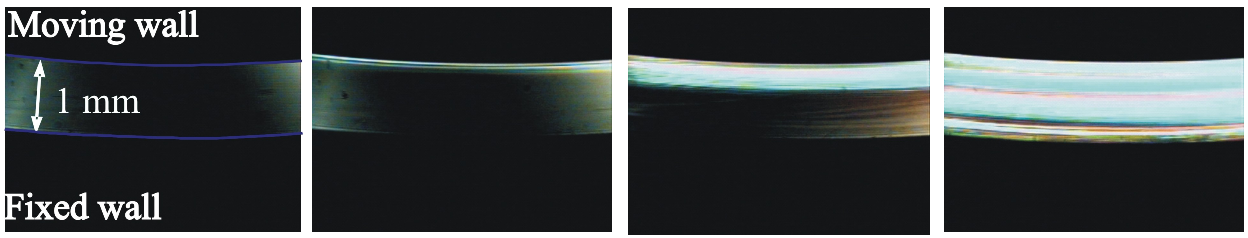
S. Lerouge



liquide viscoélastique

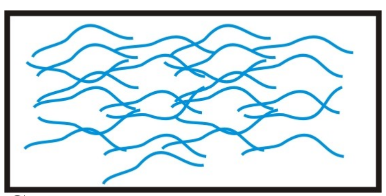
S. Lerouge

o



bandes de cisaillement

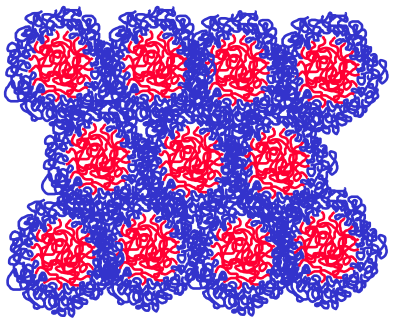
S. Lerouge

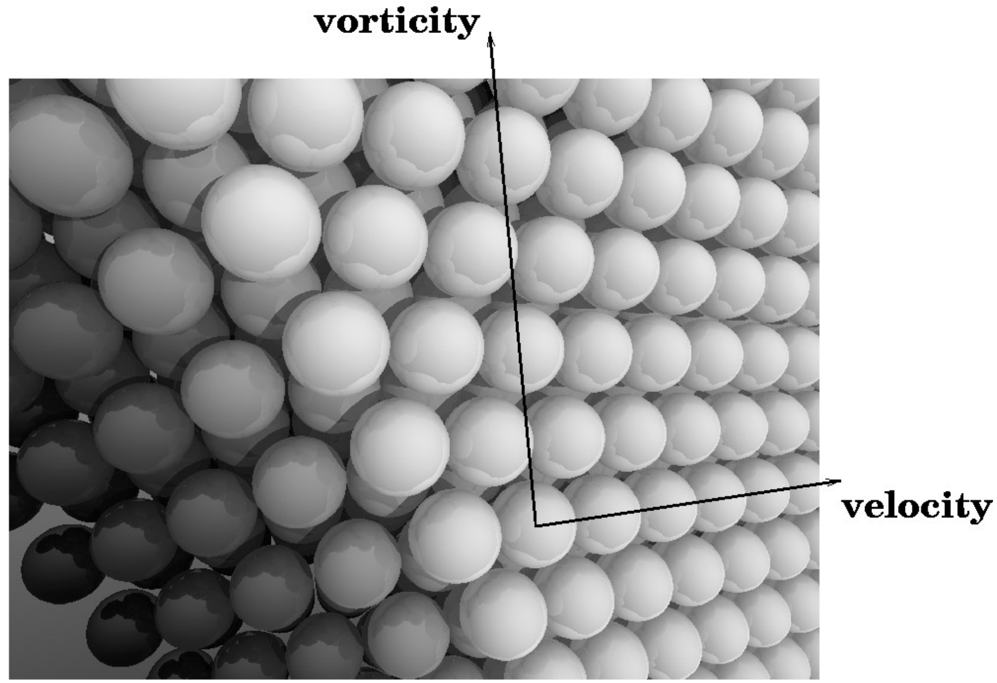


S. Lerouge

alignement → fluide → cisaillement → alignement

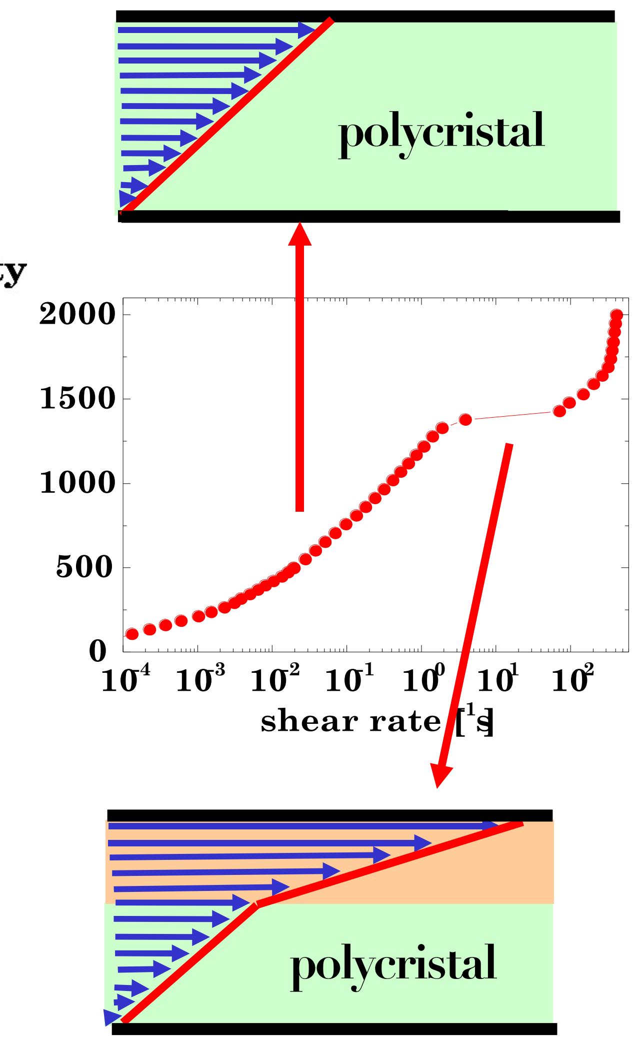
Phases cubiques de copolymères





E. Eiser, F. Molino, G. Porte, O. Diat, 2000

bandes de cisaillement

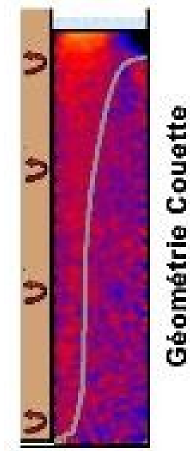


alignement → glissement entre couches → cisaillement → alignement

Granulaire

solide plastique

bandes de cisaillement



P. Snabre, B. Pouligny

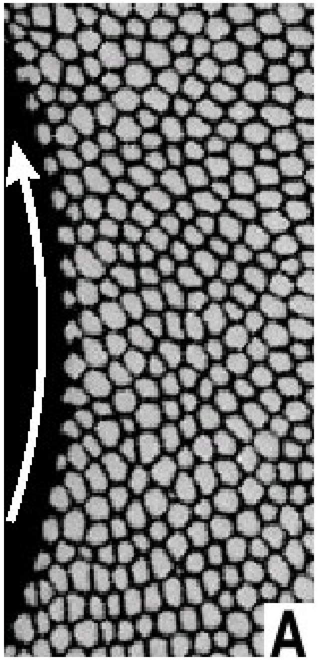
densité → fluide → cisaillement → densité ?

contacts lubrifiés

dilatance

M. Lenoble, B. Pouligny

Mousse



solide visco-élasto-plastique

bandes de cisaillement

? → fluide → cisaillement → ?

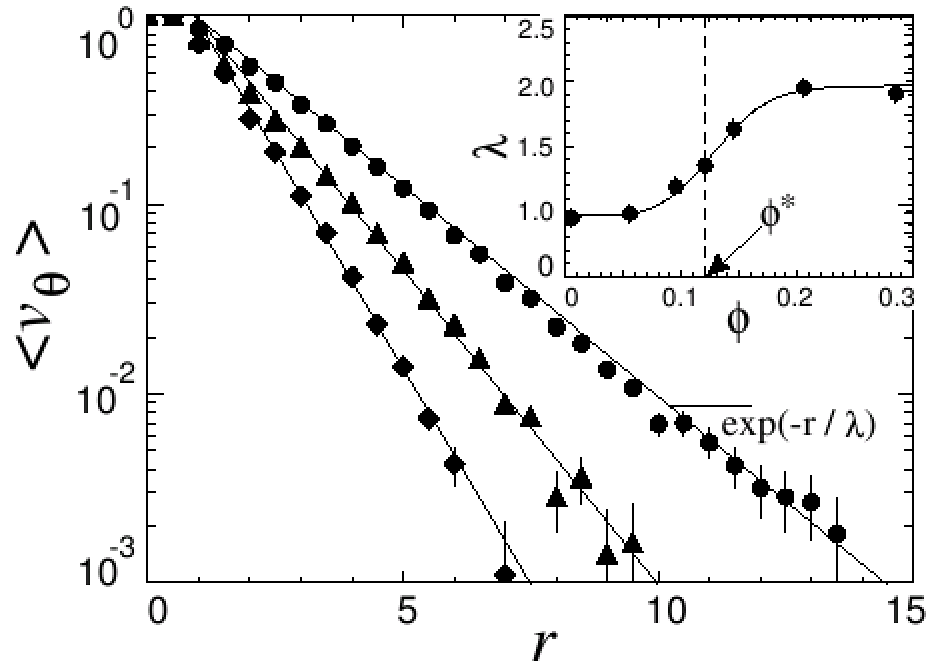
?

propagation

désordre

dilatance

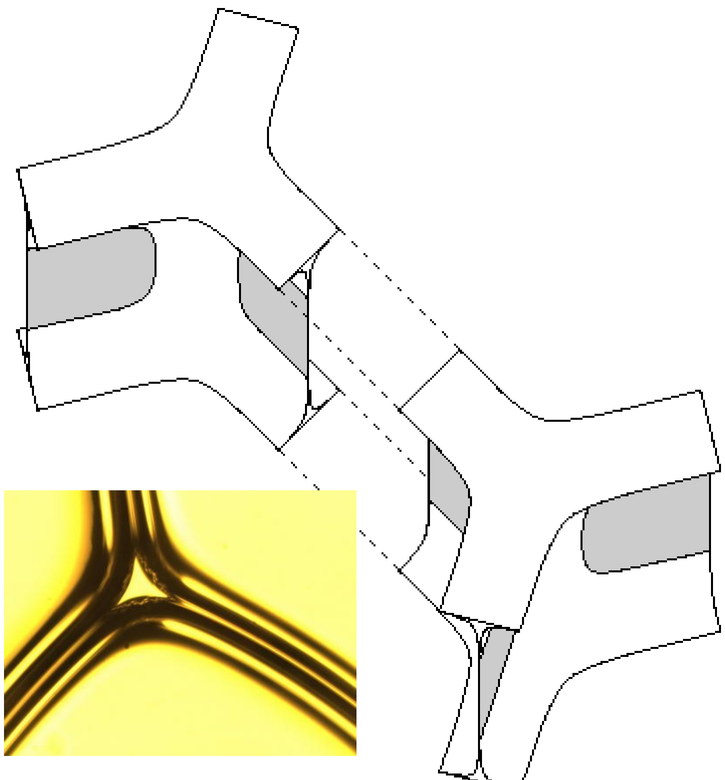
G. Debrégeas, H. Tabuteau, J.-M. Di Meglio, 2001

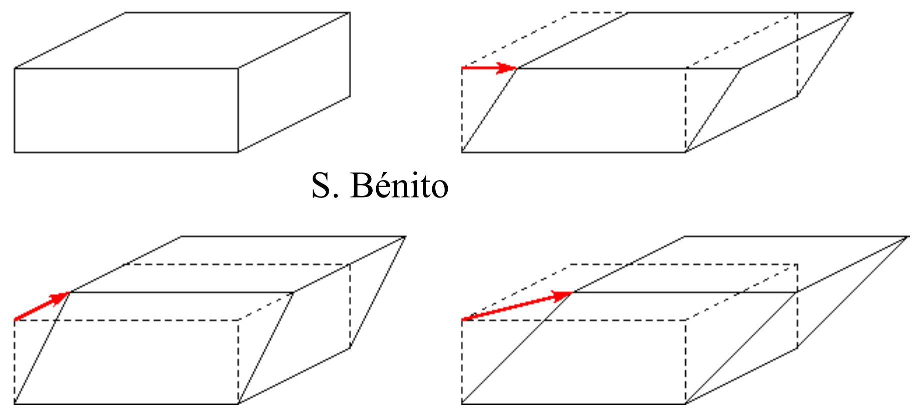


Conclusion

$$\frac{\partial e}{\partial t} + (v \cdot \nabla) e = D + \nabla v \cdot e + e \cdot \nabla v^{\mathrm{T}}$$
$$- A_0(e)\, \mathrm{I} - A_1(e)\, e - A_2(e)\, e^2$$

S. Bénito, C.-H. Bruneau, T. Colin, F. Molino





S. Bénito

Exp : C. Derec, F. Élias, M.-A. Guedeau-Boudeville, D. Osmani
Théorie : F. Molino, P. Rognon